Decomposing Multilateral Price Indexes into the Contributions of Individual Commodities

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Outline

- Decomposing bilateral indexes
  - Examples
  - Special types
- Multilateral indexes
  - Motivation
  - Decomposing a few common methods
  - Accounting for linking (“splicing”) methods
- Decomposition of multilateral indexes in practice
- Conclusions

Excerpt from Consumer Price Index, Australia, Sep 2018 (ABS)

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- Main Features
- Main Contributors to Change
- Capital Cities Comparison
- Use of Price Indexes in Contracts
- Frequently Asked Questions

MEDIA RELEASE

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CPI rose 0.4 per cent in the September quarter 2018

The Consumer Price Index (CPI) rose 0.4 per cent in the September quarter 2018, according to the latest Australian Bureau of Statistics (ABS) figures. This follows a rise of 0.4 per cent in the June quarter 2018.

The most significant rises this quarter are international holiday travel and accommodation (+4.3 per cent), tobacco (+1.8 per cent), property rates and charges (+2.3 per cent), automotive fuel (+1.4 per cent) and fruit (+2.4 per cent). The rise is partially offset by falls in child care (-11.8 per cent) and telecommunications equipment and services (-1.5 per cent).

MAIN CONTRIBUTORS TO CHANGE

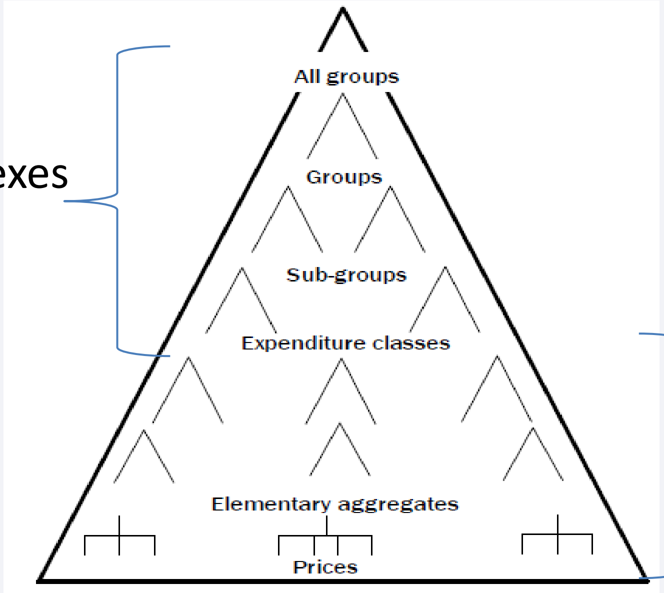
CPI GROUPS

The discussion of the CPI groups below is ordered in terms of their absolute significance to the change in All groups index points for the quarter (see Tables 6 and 7). Unless otherwise stated, the analysis is in original terms.

CPI Structure

Source: *Consumer Price Index: Concepts, Sources and Methods, 2017* (ABS)

Published indexes
Lowe method



Combination of methods

- Bilateral (including Lowe, Jevons, Dutot)
- Multilateral (CCD)

Decomposing bilateral indexes

- Studied by various authors, nice overview in Balk (2008)
- Some decompositions are straightforward...
  - Additive decomposition of Laspeyres index:

$$P_L^{0,1} = \frac{\sum_i p_i^1 q_i^0}{\sum_i p_i^0 q_i^0} = \sum_i s_i^0 \frac{p_i^1}{p_i^0}$$

  - Multiplicative decomposition of Törnqvist index:

$$P_T^{0,1} = \prod_i \left(\frac{p_i^1}{p_i^0} \right)^{\frac{1}{2}\left(s_i^0 + s_i^1\right)}$$

- Some are more complicated (e.g. Fisher)

Decomposing indexes in different ways

- From Balk (2008), multiple decompositions are possible
  - E.g. multiplicative decomposition of arithmetic mean indexes

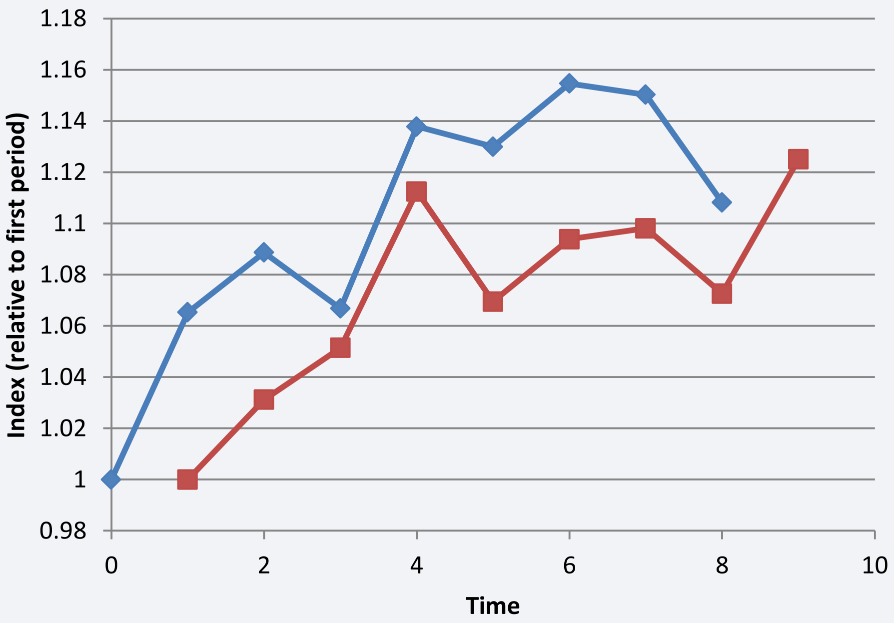
$$P^{0,1} = \sum_i w_i \frac{p_i^1}{p_i^0} = \prod_i \left(\frac{p_i^1}{p_i^0}\right)^{\sigma_i} \qquad \sigma_i = \frac{w_i \times L\left(P^{0,1}, p_i^1 / p_i^0\right)}{\sum_j w_j \times L\left(P^{0,1}, p_j^1 / p_j^0\right)}$$

- Which is preferable may depend on context
- Note σ_i depends on price index - we call decompositions with this feature *reflexive*
- Contrast with *simple* decompositions where contributions depend on price of relevant commodity, and weights

Multilateral indexes (background)

- Simultaneously compare prices or quantities across more than two entities (countries, time periods)
- Pioneering work by Ivancic, Diewert and Fox (2011) suggests using these methods to produce temporal indexes from transactions (scanner) data
- In use at ABS and a few other statistical offices (e.g. Netherlands, New Zealand), investigated by others

Multilateral indexes (fictional example)



- Calculate indexes on moving windows of data
- Link together, avoiding revisions

Interpreting multilateral movements

- Challenges in identifying influence of individual commodities on price change
  - Dependence on historical prices
  - Commodities appearing and disappearing
  - Linking
- Focus on decomposing a few common multilateral methods…
  - Time Product Dummy
  - Geary-Khamis
  - GEKS-Törnqvist / CCD (Caves, Christensen and Diewert, 1982)
- …and accounting for the linking

Time Product Dummy (TPD) method

- Fit regression model:

$$\ln p_i^t = \alpha + \boxed{\delta^t} + \boxed{\gamma_i} + \boxed{\varepsilon_i^t}$$

Time effect

Commodity effect

Error

- Common to use expenditure shares for weighting – justification in Diewert (2005)
- Price comparisons from time effect estimates:

$$P_{TPD}^{a,b} = \frac{\exp\left(\hat{\delta}^b\right)}{\exp\left(\hat{\delta}^a\right)} = \exp\left(\hat{\delta}^b - \hat{\delta}^a\right)$$

Decomposing TPD via matrices

- Express model in matrix form

$$\ln \mathbf{p} = \boxed{\mathbf{X}} \times \boxed{\boldsymbol{\beta}} + \mathbf{e}$$

Design matrix

Vector of parameters

- Solution:

$$\boldsymbol{\beta} = \boxed{\left(\mathbf{X}^T \mathbf{W} \mathbf{X}\right)^{-1} \mathbf{X}^T \mathbf{W}} \ln \mathbf{p}$$

$$= \mathbf{A} \ln \mathbf{p}$$

W= Matrix of expenditure shares

- Parameters are sums of log prices, weighted using elements of **A** (dependent on expenditure shares)
- Yields a simple decomposition:

$$P_{TPD}^{a,b} = \prod_i \prod_t \left(p_i^t\right)^{w_i^t\{b\} - w_i^t\{a\}}$$

Decomposing TPD via Rao method

- Rao (2005) shows TPD is also solution to simultaneous equations

$$P_{TPD}^{t}=\prod_{i}\left(\frac{p_i^t}{\pi_i}\right)^{s_i^t} \qquad \pi_i=\prod_{t}\left(\frac{p_i^t}{P_{TPD}^t}\right)^{\frac{s_i^t}{\sum_u s_i^u}}$$

π_i = Reference price

- Yields another decomposition

$$P_{TPD}^{a,b}=\prod_{i}\frac{\left(p_i^b\right)^{s_i^b}}{\left(p_i^a\right)^{s_i^a}}\left(\pi_i\right)^{s_i^a-s_i^b}$$

- Not new – but we note it is reflexive and not unique
- Replace missing prices and shares with 1s and 0s

The GK method

- Usually written using simultaneous equations – cf TPD method

$$P_{GK}^t = \frac{\sum_i p_i^t q_i^t}{\sum_i \pi_i q_i^t} \qquad \pi_i = \frac{\sum_t p_i^t q_i^t \big/ P_{GK}^t}{\sum_t q_i^t}$$

π_i = Reference price

- For our purposes, helpful to rewrite

$$P_{GK}^t = \sum_i \sigma_i^t \frac{p_i^t}{\pi_i} \qquad \sigma_i^t = \frac{\pi_i q_i^t}{\sum_j \pi_j q_j^t}$$

Shares (in reference price terms)

Decomposing GK

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- Tempting to take ratios of sums… but problematic if sample is dynamic

$$P_{GK}^{a,b} = \frac{\sum_{i\in b} \sigma_i^b \frac{p_i^b}{\pi_i}}{\sum_{i\in a} \sigma_i^a \frac{p_i^a}{\pi_i}}$$

- Better to convert arithmetic to geometric mean

$$P_{GK}^t = \prod_i \left(\frac{p_i^t}{\pi_i}\right)^{\theta_i^t} \qquad \theta_i^t \approx s_i^t$$

- Yields reflexive decomposition similar to TPD

$$P_{GK}^{a,b} = \prod_i \frac{\left(p_i^b\right)^{\theta_i^b}}{\left(p_i^a\right)^{\theta_i^a}} \left(\pi_i\right)^{\theta_i^a - \theta_i^b} \qquad P_{TPD}^{a,b} = \prod_i \frac{\left(p_i^b\right)^{s_i^b}}{\left(p_i^a\right)^{s_i^a}} \left(\pi_i\right)^{s_i^a - s_i^b}$$

CCD and GEKS methods

- Calculate bilateral comparisons between each pair of periods in the window and combine as follows

$$P^{a,b} = \prod_t \left(\frac{P^{t,b}}{P^{t,a}} \right)^{\frac{1}{T+1}}$$

- Choice of bilateral index
  - Fisher (RHS) leads to GEKS (LHS)
  - Törnqvist (RHS) leads to CCD (LHS) – method used at ABS
  - In practice we estimate “maximum overlap” bilateral comparisons (different contributors to different comparisons)

Decomposing CCD

- Substitute Törnqvist decomposition into CCD formula
- Yields a simple decomposition

$$P_{CCD}^{a,b} = \prod_i \frac{\left(p_i^b\right)^{w_i(\bullet,b)}}{\left(p_i^a\right)^{w_i(\bullet,a)}} \left[\prod_t \left(p_i^t\right)^{\frac{w_i(t,a)-w_i(t,b)}{T+1}} \right]$$

$w_i(s,t)$= Törnqvist weight for bilateral comparison from *s* to *t*
$w_i(\bullet,t)$=average over comparisons involving *t*

- Could decompose GEKS in analogous way (note Fisher decompositions are reflexive)

Decomposing CCD (2)

- For static sample, simplifies to CCDI index (Diewert and Fox, 2017):

$$P_{CCD}^{a,b} = \prod_i \frac{\left(p_i^b\right)^{\frac{1}{2}\left(s_i^{\bullet}+s_i^b\right)}}{\left(p_i^a\right)^{\frac{1}{2}\left(s_i^{\bullet}+s_i^a\right)}} \left(p_i^{\bullet}\right)^{\frac{1}{2}\left(s_i^a-s_i^b\right)}$$

$s_i^{\bullet}$ = Average expenditure share

$p_i^{\bullet}$ = Average price

- Factors into average of local price change and TPD-like contribution (Chessa, Verburg and Willenborg, 2017)

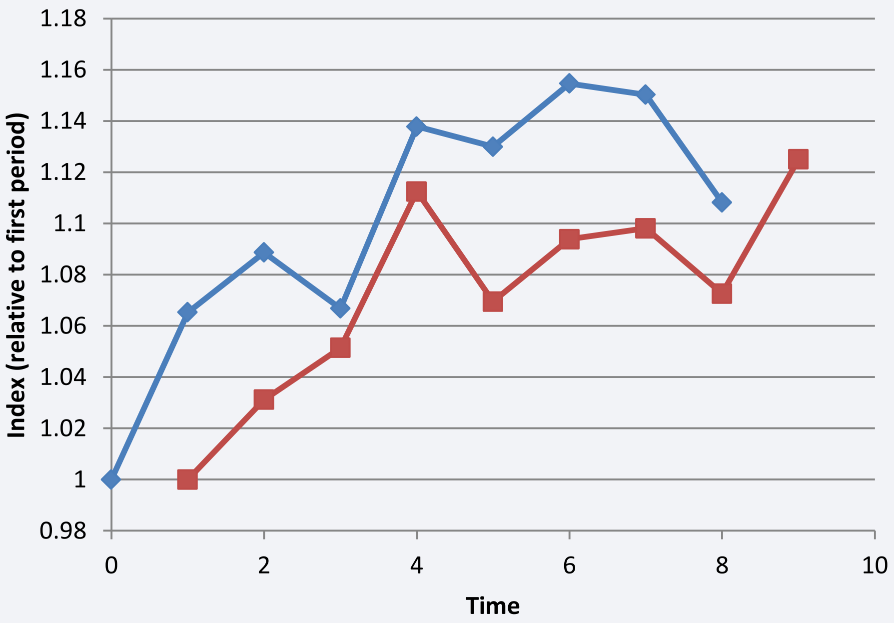
$$P_{CCD}^{a,b} = \prod_i \left[\left(\frac{p_i^b}{p_i^a}\right)^{s_i^{\bullet}}\right]^{\frac{1}{2}} \left[\frac{\left(p_i^b\right)^{s_i^b}}{\left(p_i^a\right)^{s_i^a}} \left(p_i^{\bullet}\right)^{\left(s_i^a-s_i^b\right)}\right]^{\frac{1}{2}}$$

$$P_{TPD}^{a,b} = \prod_i \frac{\left(p_i^b\right)^{s_i^b}}{\left(p_i^a\right)^{s_i^a}} \left(\pi_i\right)^{s_i^a-s_i^b}$$

Recap – decomposing multilateral indexes

- Multiplicative decompositions of a few indexes
  - CCD – simple
  - GK – reflexive
  - TPD – both
- Reveal strong similarities between methods (CCD a little different)
- Deal with dynamic samples symmetrically
  - Replace weighted missing prices with 1s

Extending multilateral indexes through linking ("splicing")



- Various methods, details not given here
- Key point: most methods express short term movements using ratios of multilateral movements

$$P^{8,9} = \frac{P^{b,9}_{Multilateral,current}}{P^{b,8}_{Multilateral,previous}}$$

Decomposing extended indexes

- Multiplicative decompositions and ratios work well together
  - Substitute multilateral decomposition and rearrange to obtain multiplicative decomposition
  - Deal with dynamic sample as before
  - Preserves simple property
- Can use similar technique to decompose longer term index movements (e.g. annual changes)

Decomposition in practice

- Illustrate these methods using a dataset of Fruit commodities
  - Originally from Turvey (1979) modified version in CPI manual (ILO, 2004)
  - Five commodities sold over four years
  - Some only sold for part of the year (strongly seasonal)
  - Sharp fluctuations in prices and quantities
- Decomposition results highlight a few interesting features of the methods

Prices and contributions of seasonal commodities

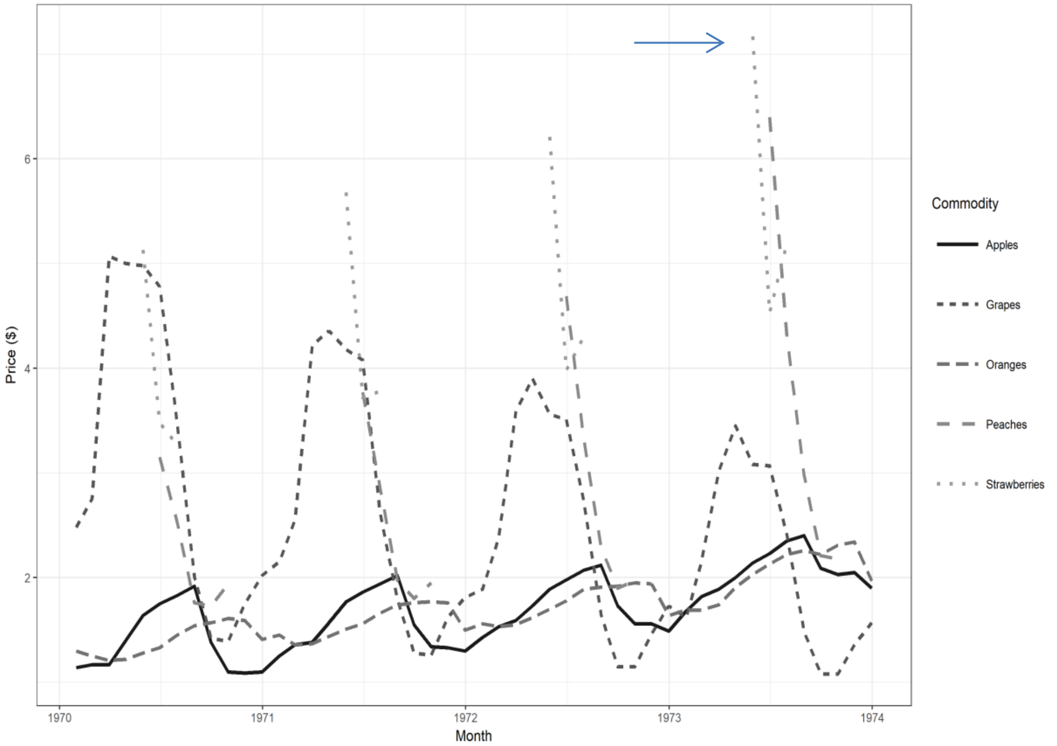


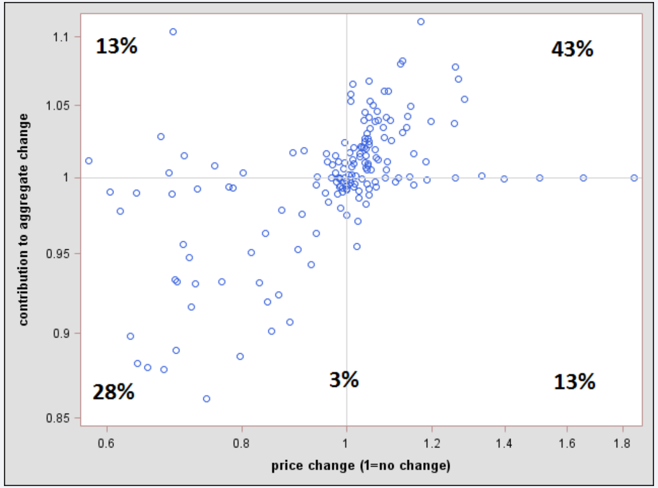
Figure 1: Price of fruit commodities

- Decomposition reveals how reappearance of seasonal commodities can contribute to price increase
- Common for commodities to have intermittent sales in transactions datasets

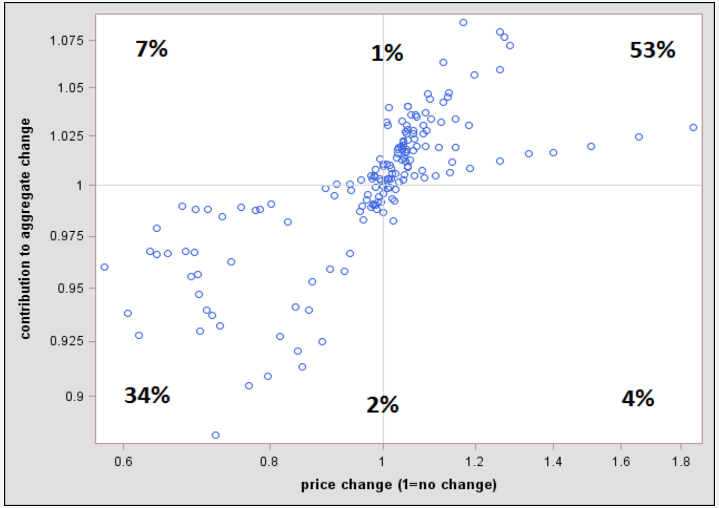
Price changes vs contributions

- Not always in the same direction!

TPD



CCD



- Stronger correlation for CCD than TPD (simple method)

Simple vs reflexive decompositions

- What happens when we tweak one of the prices of one commodity?
  - If decomposition is simple, only alters contribution of that commodity
  - If decomposition is reflexive, other commodities can also be affected
- Testing confirms this

In conclusion…

- Presented methods for decomposing multilateral indexes
  - We can use these to quantify / order contributions to change
  - Recommend using simple and symmetric methods where available
  - Formulas somewhat involved (but computers can handle the calculations)
- Theoretical and empirical results reveal
  - Similarities and differences between multilateral methods
  - Features not shared by bilateral indexes
- Avenues for further study
  - Decomposition of other indexes
  - Desirable properties of decomposition methods

References

- ABS, 2017. *Consumer Price Index: Concepts, Sources and Methods, 2017*. Cat. no. 6461.0.
- ABS, 2018. *Consumer Price Index, Australia, Sep 2018*. Cat. no. 6401.0.
- Balk, B. M. 2008. *Price and Quantity Index Numbers: Models for Measuring Aggregate Change and Difference*. New York: Cambridge University Press.
- Caves, D. W., L. R. Christensen, and W. E. Diewert. 1982. "Multilateral Comparisons of Output, Input, and Productivity Using Superlative Index Numbers." *The Economic Journal* 92, no. 365: 73-86.
- Chessa, A., J. Verburg and L. Willenborg. 2017. "A Comparison of Price Index Methods for Scanner Data." Paper presented at the 15th meeting of the Ottawa Group, Eltville, 10-12 May 2017.
- de Haan, J., and F. Krsinich. 2014. "Scanner Data and the Treatment of Quality Change in Nonrevisable Price Indexes." *Journal of Business & Economic Statistics* 32, no. 3: 341-58.
- Diewert, W. E.. 2005. "Weighted Country Product Dummy Variable Regressions and Index Number Formulae." *Review of Income and Wealth* 51, no. 4: 561-70.
- Diewert, W. E., and K. J. Fox. 2017. "Substitution Bias in Multilateral Methods for CPI Construction using Scanner Data". Vancouver: Vancouver School of Economics, University of British Columbia.
- ILO/IMF/OECD/UNECE/Eurostat/The World Bank. 2004. *Consumer Price Index Manual: Theory and Practice*. Geneva: International Labour Organization.
- Ivancic, L., W. Erwin Diewert, and K. J. Fox. 2011. "Scanner Data, Time Aggregation and the Construction of Price Indexes." *Journal of Econometrics* 161, no. 1: 24-35.
- Rao, D. S. P. 2005. "On the Equivalence of Weighted Country-Product-Dummy (CPD) Method and the Rao-System For Multilateral Price Comparisons." *Review of Income and Wealth* 51, no. 4: 571-80.
- Turvey, R. 1979. "Treatment of Seasonal Items in Consumer Price Indices". In Bulletin of Labor Statistics, 4, 13-23. Geneva: ILO.

Thank you!

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