Trapped Wave Drag from a 2D Obstacle in a Stratified Flow: Theory & Experiments

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1. INTRODUCTION

When a stratified fluid flow encounters an obstacle, the resulting internal waves in the fluid give rise to many phenomena, including trapped waves, which propagate downstream from the obstacle and impart a drag on the flow. Orographic drag of this kind is an important parameter for weather forecast models, which parameterise the drag of unresolved hills and mountains. Here, it is demonstrated that many, widely varied trapped wave models exhibit similar behaviour, in terms of flow-fields and drag, governed by a simple set of non-dimensional numbers. These different models allow for connections to be made between widely different fluid flows, from atmospheric flows over mountain ranges, to tow-tank experiments in the laboratory.

2. DISCUSSION

Stratified flows over obstacles can be approximated with the linear Taylor-Goldstein equation [1][2]

$$\frac{\partial^2 w}{\partial z^2} + (l^2 - k^2)w = 0 \tag{1}$$

where z the vertical coordinate, w is the vertical velocity, k is the horizontal wavenumber, and l is the Scorer parameter [3], which is a function of the background flow velocity U and the background buoyancy frequency N. When l^2 decreases with height, waves can become "trapped" below a certain altitude, and can propagate downstream. A straightforward example of an analytical trapped wave field can be derived for a piecewise constant Scorer parameter, of the form:

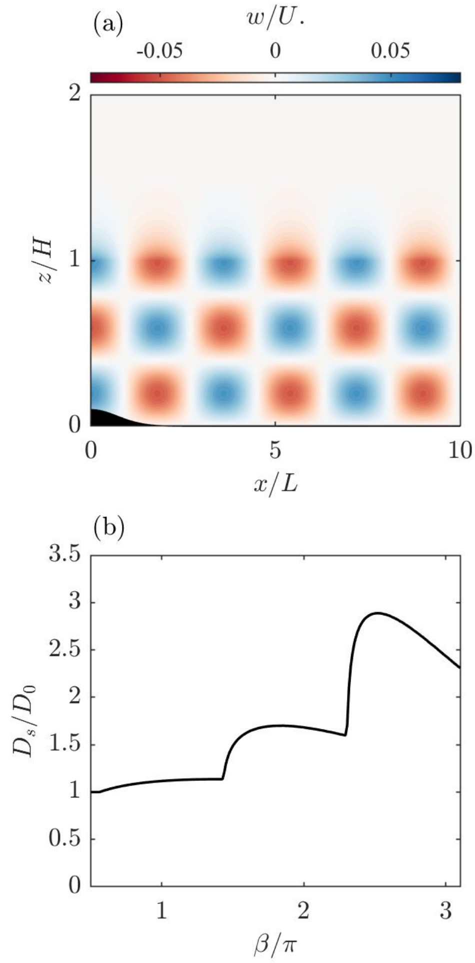

Figure 1(a) An analytical trapped wave velocity field for a Gaussian obstacle, and *(b)* Analytical wave drag for a Gaussian obstacle and different β values.

$$l^2 = \begin{cases} l_0^2, & z/H \le 1 \\ 0, & z/H > 1, \end{cases} \tag{2}$$

with l_0 and H being the magnitude and height scale of the Scorer parameter respectively. Two key non-dimensional numbers here are $\beta = l_0 H$, and $\gamma = H/L$, where L is the obstacle length scale. An analytical vertical velocity field for this l^2 distribution, with a 2D Gaussian obstacle of chord-length L, is plotted in Figure 1(a), where x is the horizontal coordinate. The plot demonstrates the limited vertical extent and distinctive "cellular" wave field that are typical of trapped waves. It will be shown that the properties of this analytical solution can be generalised to qualitatively describe the behaviour

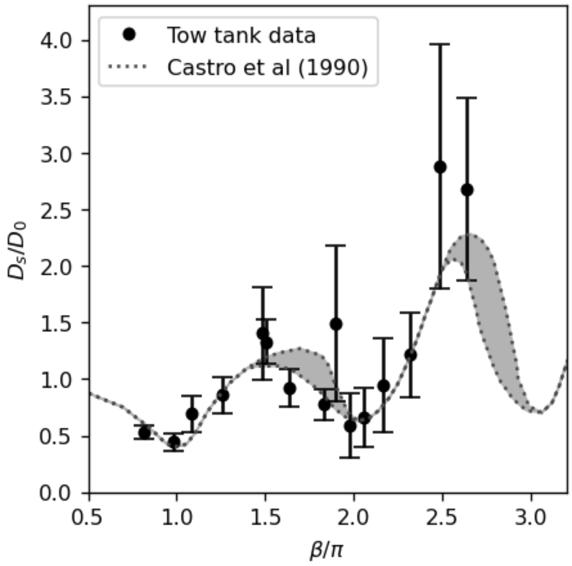

Figure 2 Experimental tow tank wave drag measurements, plotted against empirical curve from [5]

of trapped wave solutions for a variety of different Scorer parameters and boundary conditions, including exponential and linearly decreasing Scorer parameters, which more closely approximate realistic distributions in the Earth's atmosphere. The analytical solution, can also be used to obtain an equation for drag, using the method originated in Texiera eta al (2015) [4]. The analytical wave drag D_S, normalised by the unstratified turbulent form drag D_0 is plotted in Figure 1(b), showing some key properties of stratified wave drag, namely the non-monotonic increase in drag with β, and the periodic dips at regular intervals of β/π.

Drag variations, similar to those seen in Figure 1(b) can be observed experimentally with a stratified tow-tank experiment. A linear stratification profile is generated in a long tank, using water with varying salt concentrations, resulting in a fluid with a constant Scorer parameter, similar to the piecewise constant profile (2). A pseudo 2D mountain shaped obstacle is then pulled through the tow-tank at a constant speed, and drag is measured with a load cell. Stratified drag is normalised by unstratified drag to account any Reynolds number effects. An experiment of this form was conducted in Castro, Snyder & Baines (1990) [5], producing an empirical curve, shown as the grey dotted line in Figure 2. The grey areas are parameter spaces where the authors found the drag fluctuating instead of reaching a steady state. A similar experiment has now been performed with a similar obstacle, using a new tow-tank facility at the University of Melbourne. The results obtained are shown as the datapoints in Figure 2, conforming closely to the published results within the range of values tested. What we see in both cases, is drag peaking at half-integer values, and minimised at integer values of the parameter β/π, as well as a clear, but non-monotonic increase in the drag as β/π increases. The shape and features of the analytical solution do not perfectly match the analytical model, but this is to be expected, since the boundary conditions in a tow-tank experiment are complicated, and do not match the simple boundary conditions of the linear analytical model. The analytical model also does not account for the reduction in form drag due to the stratification. Nevertheless, the favourable comparison between the experiments and the theory directly point to trapped waves as the source of this non-monotonic drag. In order to further test the relationship between analytical drag equation and real world drag, the experiment will be repeated using obstacles with different L values and compared to analytical model predictions.

3. CONCLUSIONS

Different analytical trapped wave solutions share key traits, which can be generalised across different Scorer parameter distributions, from a simple linear stratification to more realistic distributions. Several of these traits can be demonstrated in the laboratory using a stratified tow-tank experiment.

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