13th Australasian Natural Convection Workshop (13ANCW)
Canberra, Australia, 1st and 2nd December 2025

DNS study of the streamwise development of natural convection boundary layers for $Pr = 2$ and $Pr = 6$

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1. INTRODUCTION

Natural convection boundary layers (NCBL) commonly arise along vertical surfaces subject to heating or cooling. These thermally-induced flows are controlled by the Rayleigh number, $Ra = g\beta\Delta T L_c^3/\nu\alpha$, and the Prandtl number, $Pr = \nu/\alpha$. Here, ΔT denotes the temperature difference between the vertical surface and the ambient, β is thermal expansion, g is gravitational acceleration, ν is viscosity, α is thermal diffusivity, and L_c is a characteristic length scale. The wall heat transfer is characterised by the Nusselt number ($Nu = q_w L_c/k\Delta T$), where k is thermal conductivity, and q_w is the local wall heat flux. Evidently, the dimensionless parameters Ra and Nu depend on a local length scale, L_c, characterising the development of the flow. For a spatially developing flow, the streamwise location x is conventionally chosen as L_c, whereas the momentum boundary layer thickness δ_U is adopted for transient flows.

Previous investigations revealed that Nu scales with $Ra^{1/3}$ in the turbulent regime [1–3], but more recent studies suggest this exponent could deviate from $1/3$ at higher Ra [4]. However, existing measurements for spatially developing NCBL are limited due to physical laboratory constraints [1,2], and computational capabilities [3]. Recently, direct numerical simulations (DNS) have been performed in a temporal framework to efficiently obtain high Ra with reduced computational requirements [5,6]. Comparisons of transient and spatially developing NCBL obtained by empirically matching the δ_U show good agreement in the turbulent statistics, suggesting the transient and spatially developing flows may share the same physics. However, the absence of a Pr-dependent scaling relation linking boundary layer thickness δ_U to streamwise location x limits *a priori* comparisons between these flows. The present study considers a spatially developing and steady NCBL up to $Ra_x \sim 10^{11}$ with $Pr = 2$ and $Pr = 6$. Using DNS, we obtain the evolution of the momentum and thermal boundary layer thicknesses with streamwise location to enable a systematic comparison with transient NCBL.

2. NUMERICAL METHOD

The governing equations are the incompressible continuity, Navier-Stokes, and energy conservation equations with the Oberbeck–Boussinesq approximation. The equations are discretised in a Cartesian system with domain sizes of $(L, 0.278L, 0.1L)$ for $Pr = 6$ and $(L, 0.347L, 0.111L)$ for $Pr = 2$. Uniform mesh is applied in the streamwise (x) and spanwise (z) directions, while a log-growth mesh is implemented in the wall-normal (y) direction. A $2400 \times 240 \times 350$ grid is used for $Pr = 6$, and an $1800 \times 200 \times 240$ grid is used for $Pr = 2$. For both cases, a zero-Neumann condition is enforced on all flow variables to the top outlet and the far field. The heated surface is isothermal ($T_w = 1$) and non-slip. The spanwise boundaries are periodic. The working fluid is initially at a temperature $T = 0$ and quiescent. To excite the turbulent transition, artificial broadband perturbations are introduced: For $Pr = 6$, a random temperature fluctuation is imposed on the heated surface with a magnitude of $0.1\Delta T$ in the region of $x \in (0, 0.083L)$; whereas for $Pr = 2$, a uniform random number generator is added to the source term in the energy equation in the region of $x \in (0.028L, 0.056L)$ and $y \in (0.0014L, 0.014L)$.

Additionally, a damping force is applied near the top boundary, $x \in (L, 1.111L)$, to prevent reflective influence. A buffer region is applied at the bottom, $x \in (-0.083L, 0)$, to limit the influence of bottom geometry.

3. RESULTS

Momentum (δ_U) and thermal (δ_T) boundary layer thicknesses are defined respectively as:

$$\delta_U = \int_0^\infty U/U_{max} dy, \quad \delta_T = \int_0^\infty T/(T - T_w) dy. \tag{1}$$

The evolutions of the momentum and temperature boundary layer thicknesses are presented in Fig. 1. Evidently, δ_U is always larger than δ_T, and both δ_U and δ_T decrease with increasing Pr at high Ra_x. In the fully turbulent regime, least-squares fits of δ_U amd δ_T give distinct power-law correlations for both quantities at $Pr = 2$ and $Pr = 6$:

$$\delta_U/L \approx 0.032 \times 10^{-7} Ra_x^{0.628}, \quad \delta_T/L \approx 0.093 \times 10^{-5} Ra_x^{0.326}, \quad Pr = 2, \tag{2a}$$

$$\delta_U/L \approx 0.239 \times 10^{-7} Ra_x^{0.539}, \quad \delta_T/L \approx 0.402 \times 10^{-5} Ra_x^{0.253}, \quad Pr = 6. \tag{2b}$$

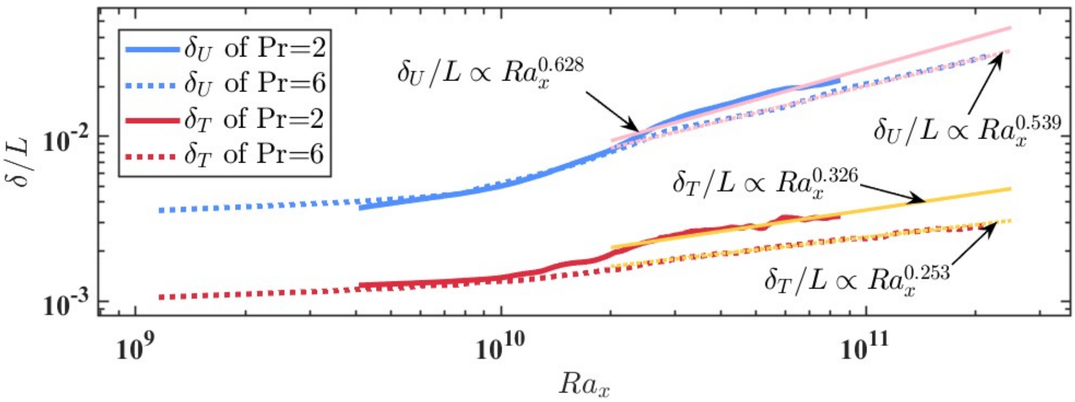

Figure 1. Boundary layer thickness streamwise development

4. CONCLUSIONS

DNS has been conducted for the spatially developing NCBL with $Pr = 2$ and $Pr = 6$ up to $Ra_x \sim 10^{11}$. The results suggest that the streamwise evolution of the boundary layer thickness can be approximated by a Pr-dependent power law, where a higher Pr leads to a lower boundary layer growth rate in x in the turbulent regime.

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