13th Australasian Natural Convection Workshop (13ANCW)
Canberra, Australia, 1st and 2nd December 2025

On the role of Prandtl number in vertical natural convection

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1. INTRODUCTION

Vertical natural convection boundary layers (NCBL) are ubiquitous in a wide range of geophysical and engineering applications. These thermally driven flows have been extensively studied since the 1930s [1], where efforts have primarily focused on understanding the flow dynamics at specific Prandtl numbers (Pr), particularly for air ($Pr \approx 0.7$) [2,3] and water ($Pr = 4 \sim 6$) [4,5]. These studies highlight evident and distinct flow behaviors at different Prandtl numbers, suggesting that Pr plays a vital role in shaping both thermal and momentum boundary layer structures. Such Prandtl number effects span across the laminar, transition and turbulent regimes, affecting not only the boundary layer development but also the transition onset [6] and turbulent transfer [7]. There remains a lack of systematic investigations that enables a direct comparison of the flows at matched configurations to quantify the influence of Prandtl number. In the present study, we investigate the Prandtl number effects on the development of vertical NCBL by employing direct numerical simulations (DNS) for $Pr = 4.16$ and $Pr = 6$ in a temporally developing configuration. These Prandtl numbers correspond to water at 42^oC and 25^oC, respectively. The statistics are compared with the DNS data reported in [8] for $Pr = 0.71$ to enable a direct comparison of the Prandtl number in a wider range.

2. DIRECT NUMERICAL SIMULATION

The numerical setup follows that of the $Pr = 0.71$ case described in [8], where the flow is considered spatially invariant by imposing periodic boundary conditions in the streamwise (x) and spanwise (z) directions. In this setup, the temporally evolving flow is homogeneous in the x–z plane and the mean statistics (denoted by $\overline{(\cdot)}$) are obtained by averaging over this plane at a given time instant t and wall-normal location y. To excite laminar–turbulent transition, a temperature perturbation was added to the laminar initial condition so that both thermal and momentum field will respond directly. In the present study, the laminar flows are initialised using the analytical solutions [9] at arbitrarily chosen time t (and therefore initial Rayleigh number) with a fixed-amplitude broadband white noise for all Pr cases so that the differences in the flow behaviors are attributed to intrinsic Prandtl number effects rather than perturbations.

3. RESULTS

Figure 1 shows the development of the dimensionless wall heat transfer, Nusselt number, with the Rayleigh number, defined by

$$Nu_\delta = \frac{\delta}{\theta_w}\left(\frac{\partial\overline{\theta}}{\partial y}\right)_w, \qquad Ra_\delta = \frac{g\beta\theta_w\delta^3}{\nu\alpha}, \qquad \delta = \int_0^\infty \frac{\overline{u}}{\overline{u}_m}\mathrm{d}y, \tag{1a,b,c}$$

where $\theta = T - T_\infty$ is the local temperature difference, g the gravitational acceleration, β denotes the thermal expansion coefficient, ν is the viscosity and α is the thermal diffusivity — their ratio gives the Prandtl number $Pr = \nu/\alpha$. In equation (1c), δ is the momentum integral thickness of the boundary

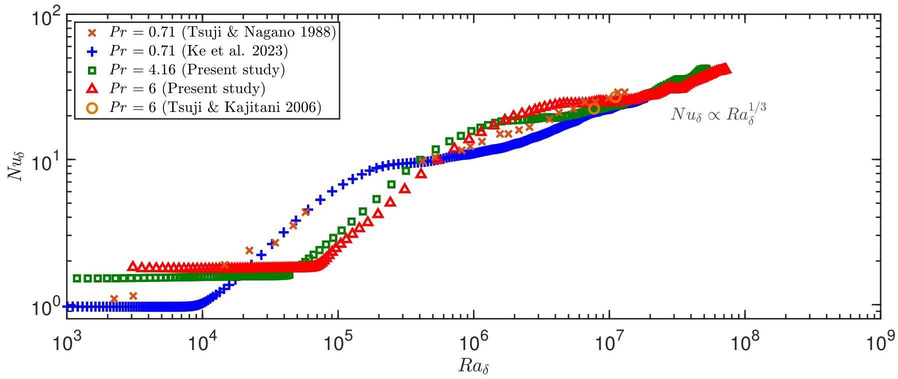

Figure 1. Nusselt number development of the NCBL at $Pr = 0.71$, 4.16 and 6.

layer and $\overline{u}_m(t)$ is the instantaneous maximum of the mean streamwise velocity $\overline{u}$ at time t. As shown in figure 1, in the laminar regime the Nusselt numbers are constant with evident Pr dependencies. As the flow continues to develop, a clear deviation from the constant Nu_δ values in the laminar regime is seen for all case considered. With increasing Pr, the Nu_δ becomes larger. Such a deviation indicates the onset of laminar–turbulent transition, and the critical Rayleigh number marking this onset is seen to increase systematically with Pr for the data presented. In the transitional regime, all Pr cases show qualitatively similar trends, with an offset in the Nu_δ due to their historical laminar and onset differences. As the flow develops further into the turbulent regime, all Pr cases converge toward an empirical 1/3-power-law scaling as seen in the classical spatially developing studies [2, 5].

4. CONCLUSIONS

In this study, DNS are carried out for temporally developing vertical NCBL at $Pr = 4.16$ and $Pr = 6$ under matched configurations to enable a systematic comparison of the Pr effects. Results show that the wall heat transfer of the NCBL flow, characterised by the Nusselt number Nu_δ, show a clear Pr dependence in the laminar regime. This Pr effect further affects the onset of the laminar–turbulent transition, with higher Pr the onset delays to a larger Rayleigh number. In the turbulent regime, all Pr considered converge to the classical $1/3$-power-law as seen in spatially developing flows.

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