Seasonal Product Price Indexes: What is the “Truth”?

by Erwin Diewert
University of British Columbia and
University of New South Wales,
Yoel Finkel and Doron Sayag,
Central Bureau of Statistics, Israel.

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Introduction (cont)

- **The truth is that there is no single price index that is suitable for all purposes when we have strongly seasonal products.**
- **A strongly seasonal product or commodity is one that is only available during some seasons of the year.**
- **There are 3 distinct types of index that are suitable for different purposes in the strongly seasonal commodity context:**

(i) Year over Year Monthly Indexes; this type of index measures inflation from say October 2020 to October 2021.

(ii) Month to Month indexes; this is the type of index that is produced by National Statistical Offices and is perhaps of most interest to the public and central banks.

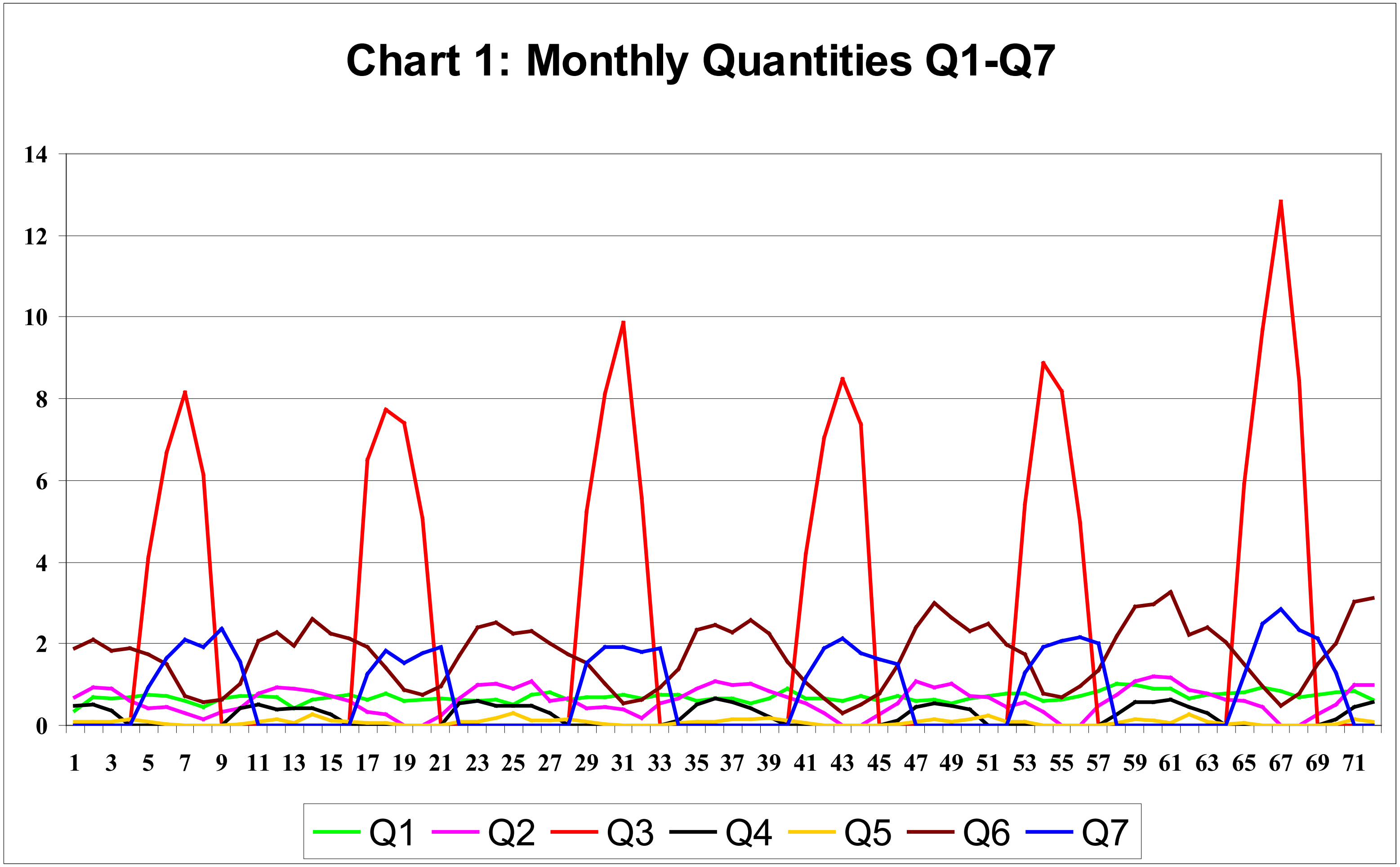
(iii) Annual Price Indexes; this type of index is needed to deflate annual nominal aggregate consumption (and its components) so that annual real consumption can be estimated.

Introduction (cont)

- **We will illustrate the above 3 types of index using monthly unit value and quantity data for the consumption of 14 types of fresh fruits in Israel over the 72 months in 2012-2017.**
- **The 14 types of fruit are as follows:**
- **1 = Lemons (available in all months)**
- **2 = Avocados**
- **3 = Watermelon**
- **4 = Persimmon**
- **5 = Grapefruit**
- **6 = Bananas (available in all months)**
- **7 = Peaches**
- **8 = Strawberries**
- **9 = Cherries**
- **10 = Apricots**
- **11 = Plums**
- **12 = Clementines**
- **13 = Kiwi fruit**
- **14 = Mangos.**
- **There were 451 missing products out of 14x72=1008 possible product prices.**

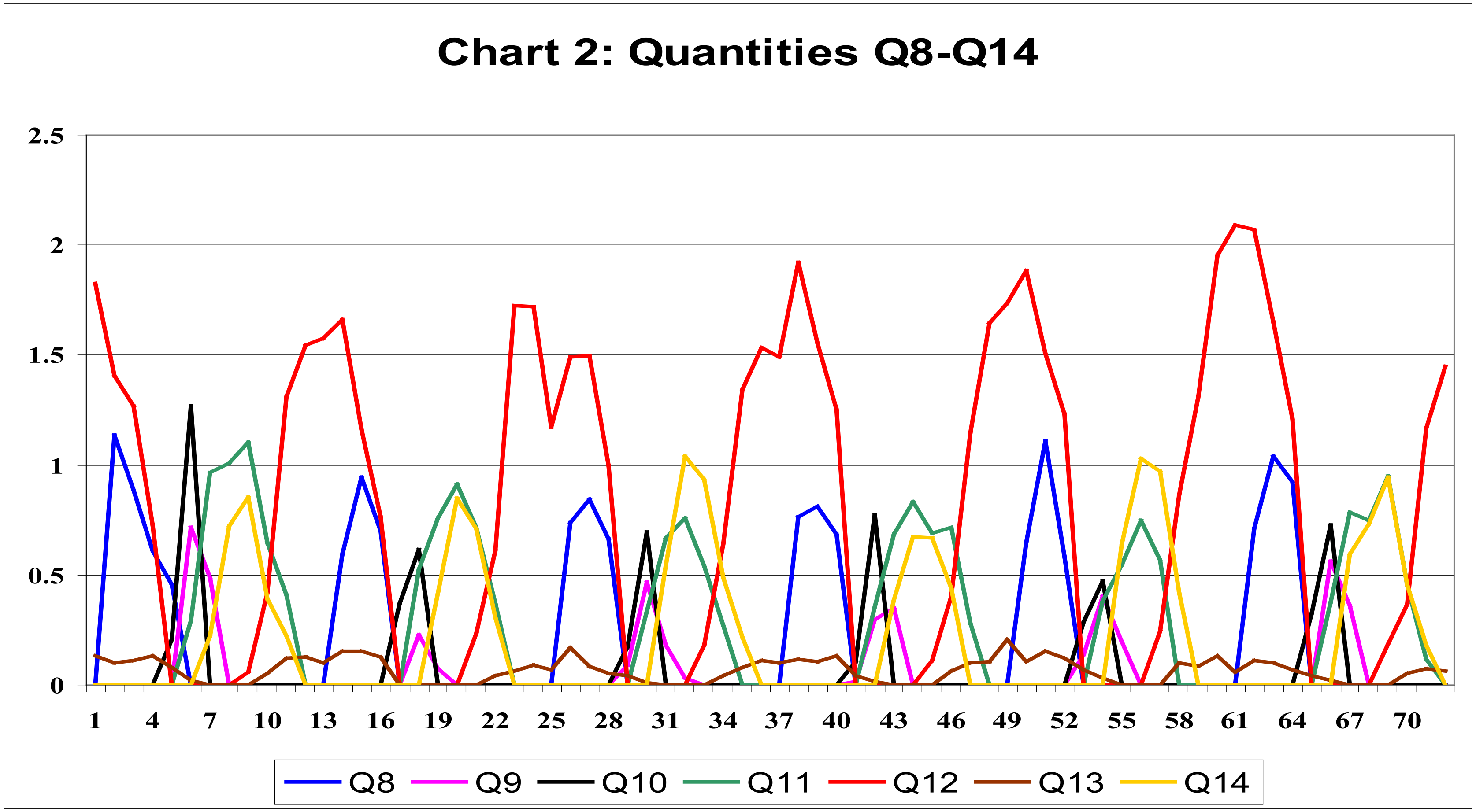
The Quantity Data: Q1-Q7

- **Here are the quantities for fruits 1-7: (Q3 = Watermelon)**



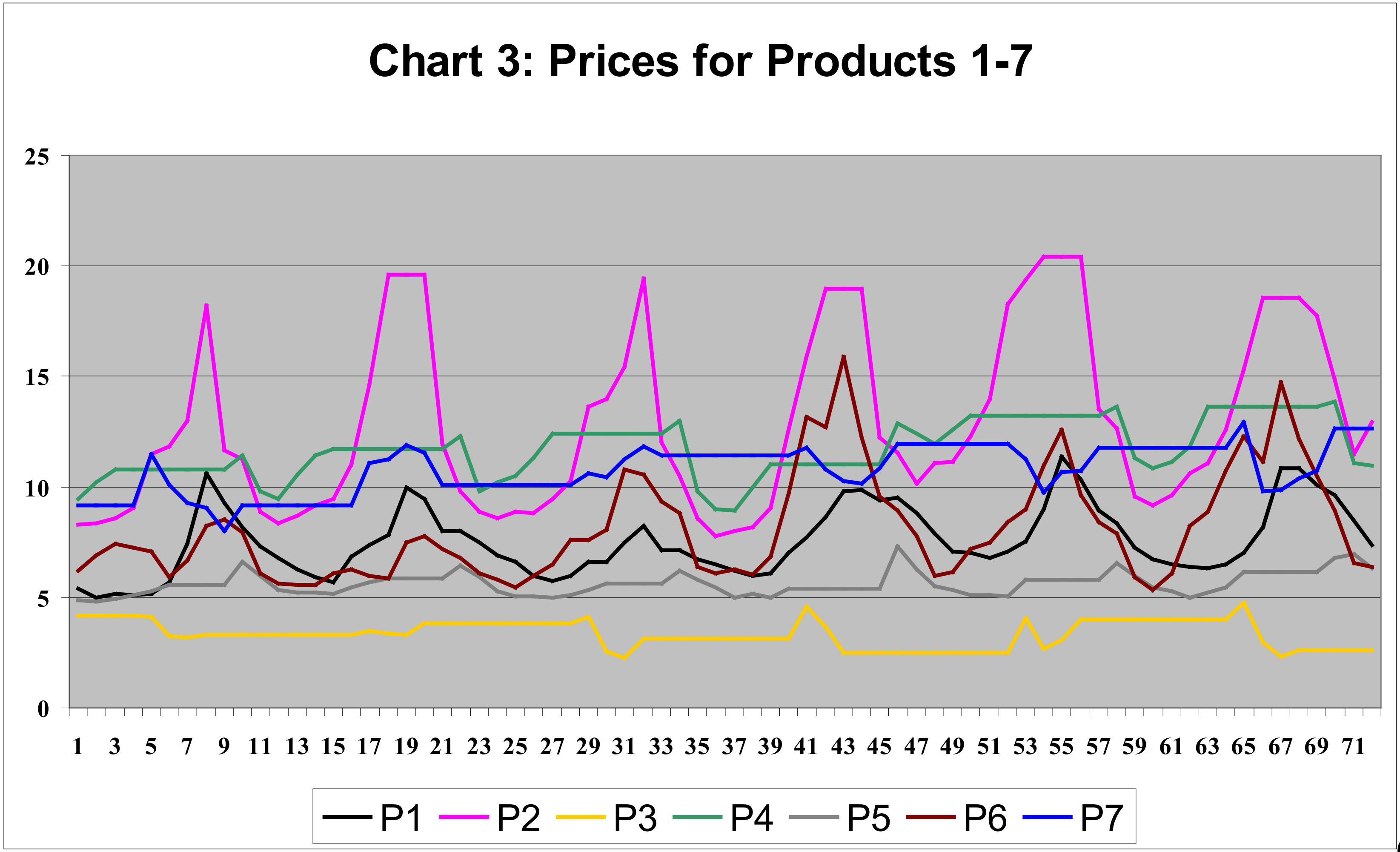
The Quantity Data (Q8-Q14)

Q12 = Clementines



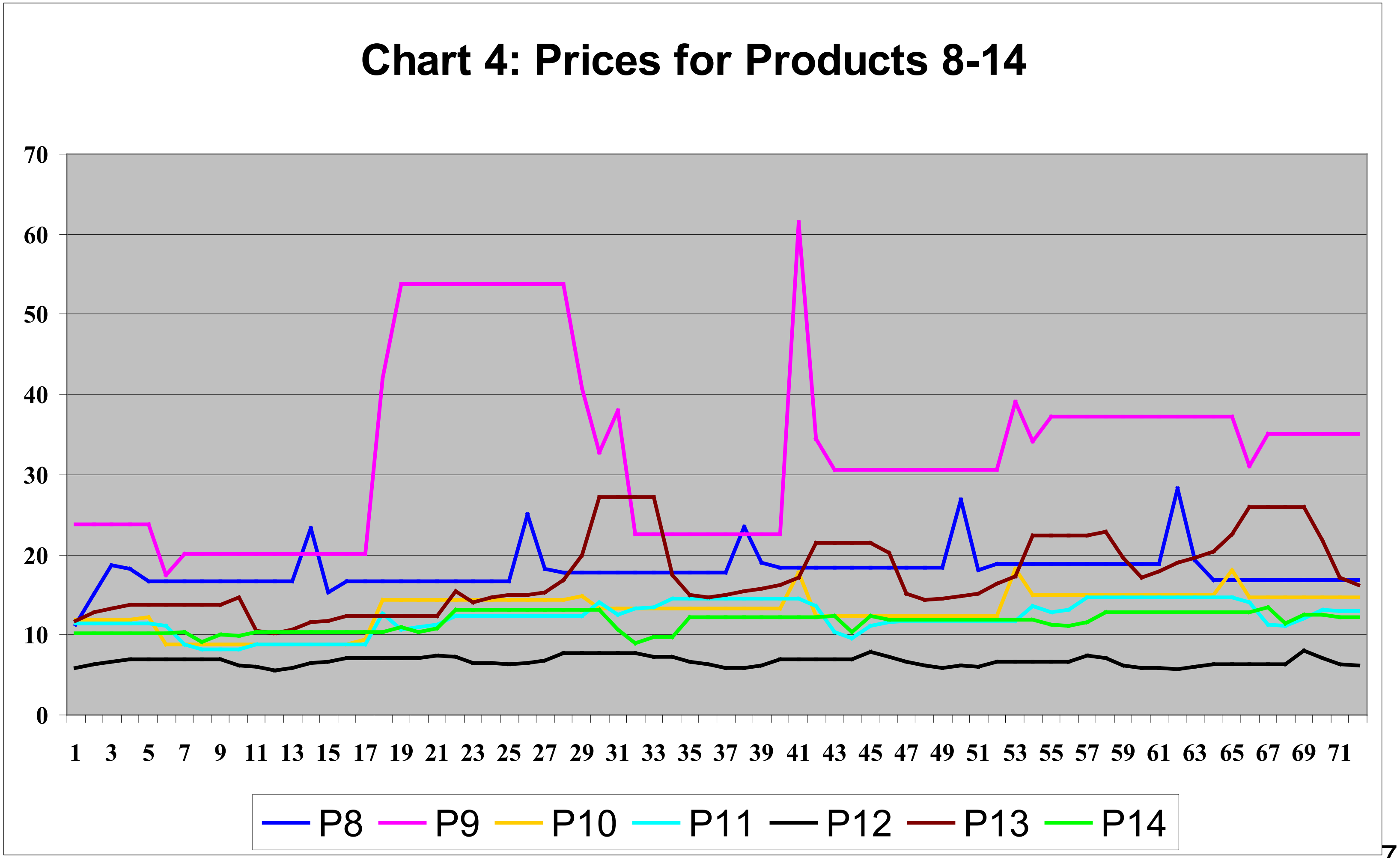
The Price Data (P1-P7)

Month to month carry forward prices are used here.



The Price Data (P8-P14)

Month to month carry forward prices are used here.



Index Number Formulae Used

P_{LFB}^t = Fixed Base Laspeyres Index using Carry Forward Prices;

P_{LCH}^t = Chained Laspeyres Index using Carry Forward Prices;

P_{PFB}^t = Fixed Base Paasche Index using Carry Forward Prices;

P_{PCH}^t = Chained Paasche Index using Carry Forward Prices;

P_{FFB}^t = Fixed Base Fisher Index using Carry Forward Prices;

P_{FCH}^t = Chained Fisher Index using Carry Forward Prices;

P_{TFB}^t = Fixed Base Törnqvist-Theil Index using Carry Forward Prices;

P_{TCH}^t = Chained Törnqvist-Theil Index using Carry Forward Prices;

P_{LFB}^{t*}, P_{LCH}^{t*}, P_{PFB}^{t*}, P_{PCH}^{t*}, P_{FFB}^{t*}, P_{FCH}^{t*}, P_{TFB}^{t*}, P_{TCH}^{t*} = Same as above except the indexes are defined only over products that are present in both periods. These indexes are maximum overlap or matched product bilateral price indexes.

Index Number Formulae Used (cont)

- P_{GEKS}^{t}, P_{GEKS}^{t*} = GEKS indexes using either carry forward prices or using maximum overlap bilateral Fisher "Star" indexes.
- These indexes are multilateral indexes.
- The Fisher star indexes take each month as the base period month and calculate all bilateral fixed base Fisher indexes for the 72 months using the same base month.
- There are 72 such sets of indexes since any one of the 72 months can serve as the base month.
- The GEKS indexes take the geometric mean of these 72 star indexes.
- P_{GEKS}^{t} uses bilateral Fisher indexes with carry forward prices for the missing prices as the basic bilateral index number formula while P_{GEKS}^{t*} uses maximum overlap bilateral Fisher indexes as the basic building block.
- Another multilateral index formula used in the paper are the Predicted Share indexes P_{S}^{t} and P_{S}^{t*}.

Predicted Share Similarity Linked Indexes

- **Similarity Linked indexes link the current period data to the prior period that has the most similar structure of relative prices.**
- **Then we use a bilateral Fisher index to link the current price and quantity data to the prior period that has minimized our measure of relative price dissimilarity.**
- **The advantage of this method over other multilateral methods is that we eliminate any possibility of chain drift; i.e., if prices this period are equal to the prices of any prior period, then our similarity linked index will register the same index level for the two periods.**
- **Walsh's multiperiod identity test will be satisfied.**
- **Problem: how to choose the measure of relative price dissimilarity?**

Predicted Share Similarity Linked Indexes (cont)

- **Various measures of the similarity or dissimilarity of relative price structures have been proposed by Allen and Diewert (1981), Kravis, Heston and Summers (1982; 104-106), Hill (1997) (2009), Aten and Heston (2009) and Diewert (2009).**
- **However, Hill and Timmer (2006) pointed out a problem with these measures of relative price dissimilarity: they do not take into account the *lack of matching problem*; i.e., these measures fail to recognize that bilateral comparisons of prices made over a smaller number of products are not as reliable as comparisons made over a larger number of matched products.**
- **This lack of matching problem is a big one in the context of constructing index numbers for a product category where many or most products are only available in some months of the year. In our empirical example, only about 60% of the seasonal products are available in a typical month.**

Predicted Share Similarity Linked Indexes (cont)

Notation:

- **The N dimensional price and quantity vectors for month m in year y are $p^{y,m} \equiv [p_{y,m,1},p_{y,m,2},\ldots,p_{y,m,N}]$ and $q^{y,m} \equiv [q_{y,m,1},q_{y,m,2},\ldots,q_{y,m,N}]$ for y = 1,…,Y and m = 1,…,M.**
- ***The expenditure share for product n in month m and year y is***

(20) $s_{y,m,n} \equiv p_{y,m,n}q_{y,m,n}/p^{y,m}\cdot q^{y,m}$;

$y = 1,\ldots,Y;\ m = 1,2,\ldots,M\ ;\ n = 1,2,\ldots,N$

- **where $p^{y,m}\cdot q^{y,m} \equiv \Sigma_{n=1}^{N}\, p_{y,m,n}q_{y,m,n}$ is the inner product of the vectors $p^{y,m}$ and $q^{y,m}$.**
- **Now think of using the *prices* of month m in year z and the *quantities* of month m in year y to *predict* the actual month m, year y, product n expenditure share $s_{y,m,n}$ defined by (20) for n = 1,…,N.**
- **The predicted share measure of relative price dissimilarity for two year over year months is defined on the following slide.**

Predicted Share Similarity Linked Indexes (cont)

- **The *predicted Share measure of relative price dissimilarity* between the prices of month m in year y and the prices of month m in year z, $\Delta_{PS}(p^{z,m},p^{y,m},q^{z,m},q^{y,m})$, is defined as:**

(22) $\Delta_{PS}(p^{z,m},p^{y,m},q^{z,m},q^{y,m})$

$$\equiv \Sigma_{n=1}^{N} [s_{y.m,n} - s_{z,y,m,n}]^2 + \Sigma_{n=1}^{N} [s_{z.m,n} - s_{y,z,m,n}]^2$$

$$= \Sigma_{n=1}^{N} [(p_{y,m,n}q_{y,m,n}/p^{y,m}\cdot q^{y,m}) - (p_{z,m,n}q_{y,m,n}/p^{z,m}\cdot q^{y,m})]^2$$

$$+ \Sigma_{n=1}^{N} [(p_{z,m,n}q_{z,m,n}/p^{z,m}\cdot q^{z,m}) - (p_{y,m,n}q_{z,m,n}/p^{y,m}\cdot q^{z,m})]^2.$$

- **In general, $\Delta_{PS}(p^r,p^t,q^r,q^t)$ takes on values between 0 and 2.**
- **If $\Delta_{PS}(p^r,p^t,q^r,q^t) = 0$, then it must be the case that relative prices are the same in month m of years z and y; i.e., we have $p^{z,m} = \lambda p^{y,m}$ for some $\lambda > 0$.**
- **A bigger value of $\Delta_{PS}(p^r,p^t,q^r,q^t)$ generally indicates bigger deviations from price proportionality.**
- **Note that Δ_{PS} takes into account the economic importance of each product.**

Example of Year over Year Indexes for May using Carry Forward/Backward Prices

- The month of May had 8 missing prices which were imputed by 6 carry forward prices and 2 carry backward prices.
- Products 1, 2, 3, 5, 6, 7 and 10 were always present in May.
- Products 4, 11, 12 and 14 were always missing in May.
- The remaining products 8, 9 and 13 were sometimes present in May and were sometimes absent.
- The predicted share measures of relative price dissimilarity defined by (22) for our Israeli data for the month of May are listed in the table on the next slide.
- The month m is equal to 5 (May).
- The years y and z range from 1 to 6.

Example of Year over Year Indexes for May using Carry Forward/Backward Prices (cont)

- **The optimal set of links can be summarized as follows:**

1 – 2
|
3 – 4 – 6
|
5.

m = 5	y = 1	y = 2	y = 3	y = 4	y = 5	y = 6
z = 1	0.00000	0.00617	0.00250	0.02222	0.01103	0.01324
z = 2	0.00617	0.00000	0.00578	0.02768	0.00883	0.01908
z = 3	0.00250	0.00578	0.00000	0.01226	0.00409	0.00690
z = 4	0.02222	0.02768	0.01226	0.00000	0.01060	0.00175
z = 5	0.01103	0.00883	0.00409	0.01060	0.00000	0.00810
z = 6	0.01324	0.01908	0.00690	0.00175	0.00810	0.00000

- **The above links are real time links.**
- **Going from May of 2012 to May of 2013, we use the Fisher index between these two month.**
- **For May of 2014, look down the y = 3 column for Δ_{PS}. 0.00250 is smaller than 0.00578 so we link May of year 3 to May of year 1. And so on. This is how $P_S^{y,5}$ for the years 1-6 was constructed.**

Example of Year over Year Indexes for May using Carry Forward/Backward Prices (cont)

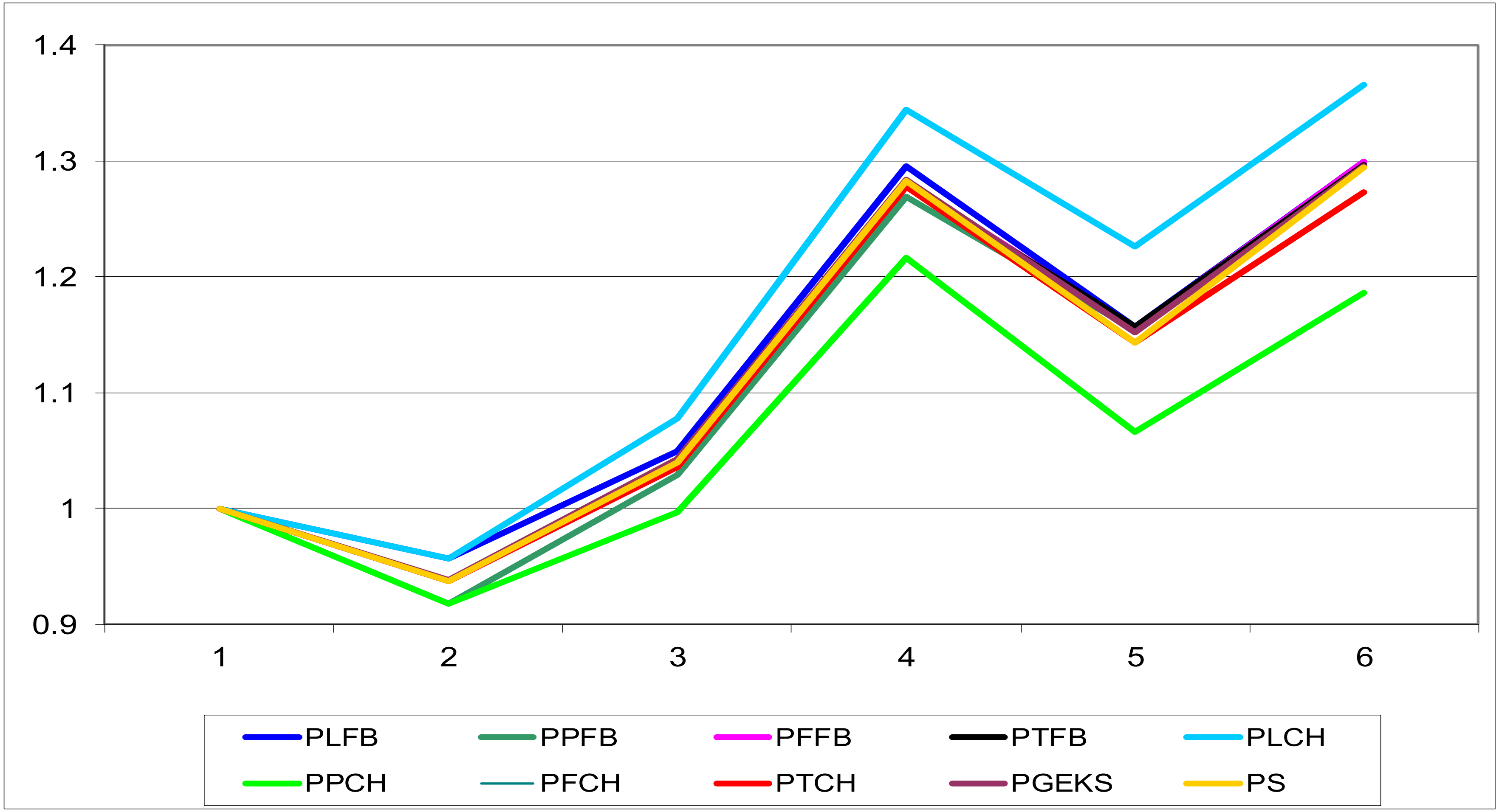
- Here is a table listing all of the various year over year indexes for May using carry forward/backward prices.

Table 4: Year over Year Alternative Indexes for May

Year y	$P_{LFB}^{y,5}$	$P_{PFB}^{y,5}$	$P_{FFB}^{y,5}$	$P_{TFB}^{y,5}$	$P_{LCH}^{y,5}$	$P_{PCH}^{y,5}$	$P_{FCH}^{y,5}$	$P_{TCH}^{y,5}$	$P_{GEKS}^{y,5}$	$P_{S}^{y,5}$
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	0.95731	0.91814	0.93752	0.93708	0.95731	0.91814	0.93752	0.93708	0.93879	0.93752
3	1.04955	1.02931	1.03938	1.03929	1.07750	0.99674	1.03634	1.03544	1.04223	1.03938
4	1.29576	1.26861	1.28211	1.27958	1.34446	1.21671	1.27899	1.27733	1.28376	1.28275
5	1.15686	1.15394	1.15540	1.15718	1.22628	1.06571	1.14318	1.14348	1.15227	1.14281
6	1.29885	1.29900	1.29893	1.29611	1.36519	1.18589	1.27239	1.27244	1.29548	1.29399
Mean	1.12640	1.11150	1.11890	1.11820	1.16180	1.06390	1.11140	1.11100	1.11880	1.11610

Example of Year over Year Indexes for May using Carry Forward/Backward Prices (cont)

- **Here is a chart for the previous table:**



Example of Year over Year Indexes for May using Carry Forward/Backward Prices: Chart Analysis

- **The Chained Laspeyres index has a lot of upward chain drift.**
- **The Chained Paasche index has a lot of downward chain drift.**
- **The Fixed Base Laspeyres index has a small amount of upward chain drift while the Fixed Base Paasche index has a small amount of downward chain drift.**
- **The Törnqvist Theil Chained index has a small amount of downward chain drift.**
- **The remaining 5 (superlative) indexes are all very close to each other.**
- **Tentative conclusion: year over year monthly fixed base superlative indexes plus the GEKS and Predicted Share Similarity linked indexes all give much the same answer using carry forward/backward prices.**
- **The following slide summarizes the results for all 12 indexes.**

Example of Year over Year Indexes for May using Maximum Overlap or Matched Model Indexes (No Imputations)

- **Now we switch to computing maximum overlap indexes instead of using carry forward prices.**
- **For the Predicted Share dissimilarity measures, if a price is missing, it is now set equal to 0 (the corresponding quantity is also set equal to 0).**
- **If the products that were purchased in month m of years y and z were identical, then the "new" measure of relative price dissimilarity defined will be identical to the "old" measure defined by (22).**
- **However, in the case where prices in years y and z are not matched, Δ_{PS} will generate a larger measure of price dissimilarity than was generated using carry forward prices; i.e., there is now a *penalty for a lack of price matching* (which can be large if the difference between $s_{y.m,n}$ and $s_{z.m,n}$ is large for an unmatched product n).**

Example of Year over Year Indexes for May using Maximum Overlap or Matched Model Indexes (No Imputations)

Table 6: May Predicted Share Measures of Price Dissimilarity Excluding Imputed Prices

m = 5	y = 1	y = 2	y = 3	y = 4	y = 5	y = 6
z = 1	0.00000	0.02471	0.02505	0.04297	0.03604	0.03227
z = 2	0.02471	0.00000	0.00988	0.02858	0.01565	0.01926
z = 3	0.02505	0.00988	0.00000	0.01226	0.00409	0.01042
z = 4	0.04297	0.02858	0.01226	0.00000	0.01060	0.00204
z = 5	0.03604	0.01565	0.00409	0.01060	0.00000	0.01445
z = 6	0.03227	0.01926	0.01042	0.00204	0.01445	0.00000

Example of Year over Year Indexes for May using Maximum Overlap or Matched Model Indexes (cont)

- The set of optimal real time bilateral links for the May data can be summarized as follows:

```
1 – 2 – 3 – 4
        |    |
        5    6.
```

- The new set of May bilateral links is different from the set of bilateral links for May that used carry forward and carry backward prices.
- Note that we simply use chained indexes for the first 4 years.
- To see the differences between the carry forward indexes for May listed in Table 4 above with the new corresponding maximum overlap indexes for May, see Table 7 on the next slide.

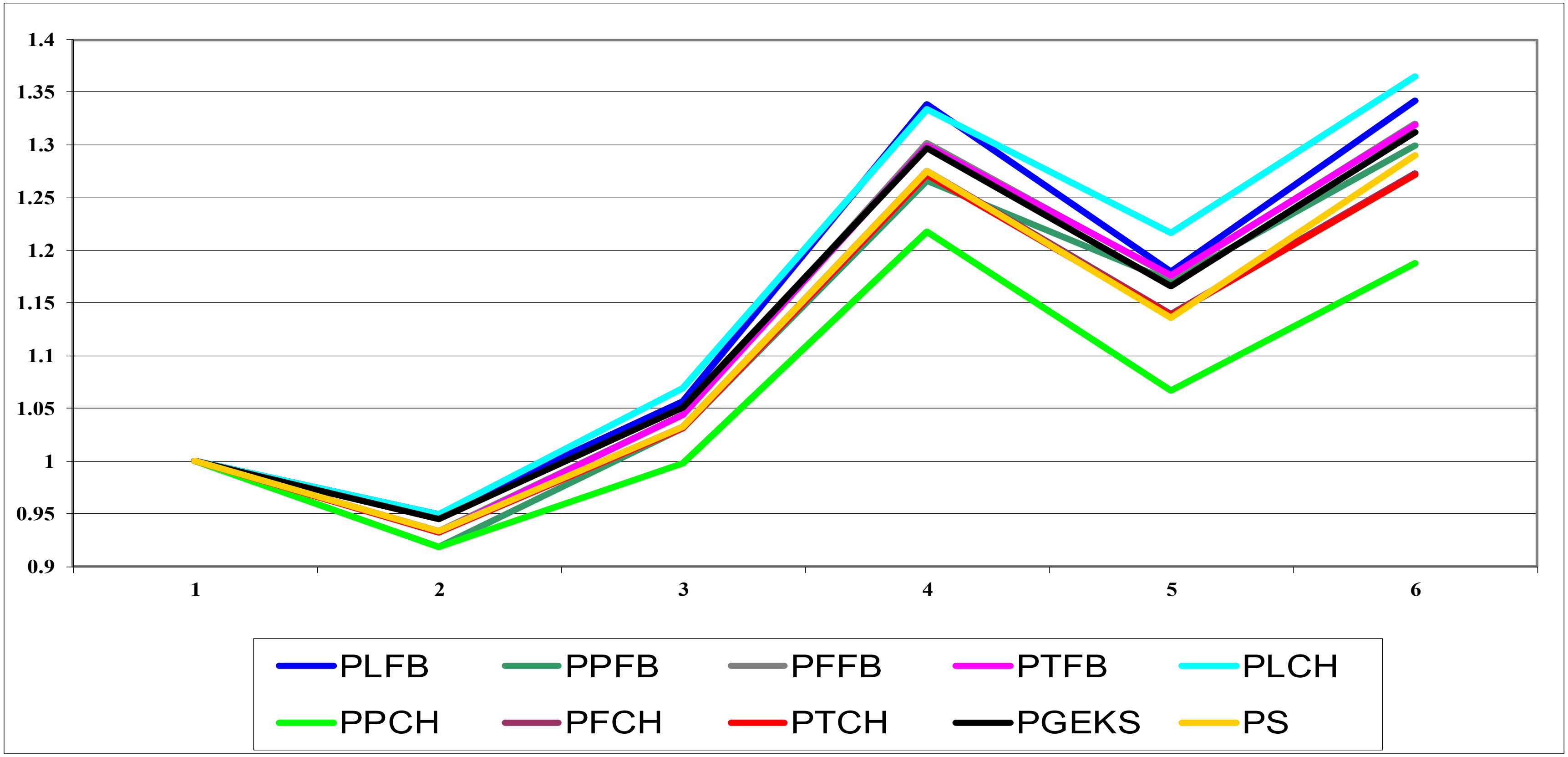
Example of Year over Year Indexes for May using Maximum Overlap or Matched Model Indexes (Table)

Table 7: Year over Year Maximum Overlap Indexes for May

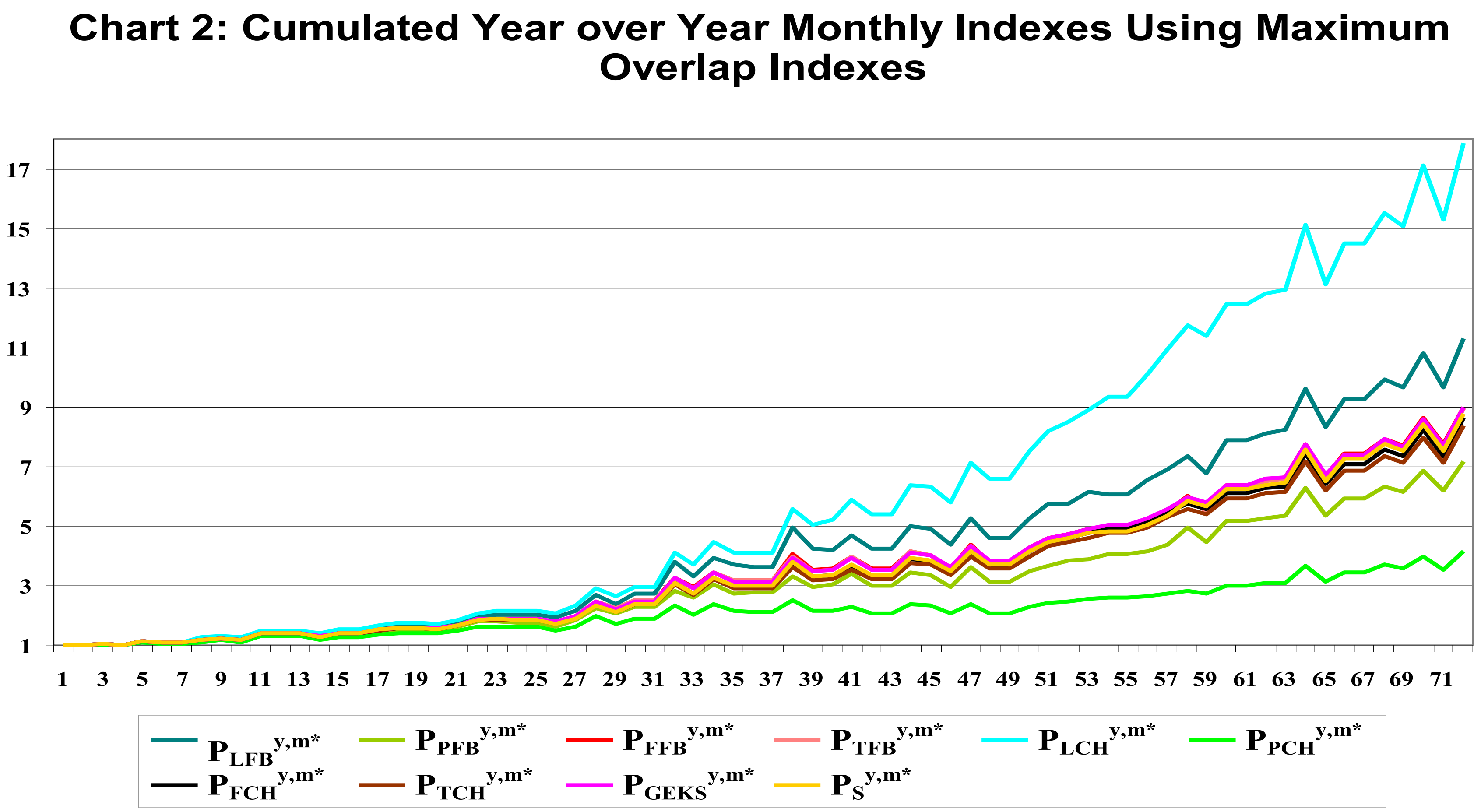
Year y	$P_{LFB}^{y,5*}$	$P_{PFB}^{y,5*}$	$P_{FFB}^{y,5*}$	$P_{TFB}^{y,5*}$	$P_{LCH}^{y,5*}$	$P_{PCH}^{y,5*}$	$P_{FCH}^{y,5*}$	$P_{TCH}^{y,5*}$	$P_{GEKS}^{y,5*}$	$P_{S}^{y,5*}$
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	0.95007	0.91814	0.93397	0.93252	0.95007	0.91814	0.93397	0.93252	0.94462	0.93397
3	1.05674	1.03102	1.04380	1.04354	1.06935	0.99802	1.03307	1.03104	1.05052	1.03307
4	1.33870	1.26554	1.30161	1.29967	1.33429	1.21827	1.27496	1.27191	1.29677	1.27496
5	1.17963	1.17093	1.17527	1.17658	1.21701	1.06707	1.13958	1.13863	1.16610	1.13587
6	1.34224	1.29900	1.32044	1.31917	1.36461	1.18740	1.27293	1.27122	1.31228	1.28980
Mean	1.14460	1.11410	1.12920	1.12860	1.15590	1.06480	1.10910	1.10760	1.12840	1.11130

Example of Year over Year Indexes for May using Maximum Overlap or Matched Model Indexes (Chart)

- **The Laspeyres Chained and Fixed Base indexes are too high.**
- **The Chained Paasche and the Chained Törnqvist are too low.**
- **The remaining indexes are pretty close to each other.**



Conclusions about Year over Year Monthly Indexes



Conclusions about Year over Year Monthly Indexes (cont)

- **The chart on the previous slide stacks up all 12 Year over Year Monthly indexes to better show the trends in our 10 indexes using maximum overlap bilateral price indexes as basic building blocks.**
- **The chart shows that the chained Laspeyres suffers from massive upward chain drift while the chained Paasche suffers from massive downward chain drift.**
- **The fixed base Laspeyres and Paasche indexes also suffer from a chain drift problem but it is not so massive.**
- **The remaining 10 indexes are clustered around each other.**
- **We prefer the similarity linked indexes since they cannot suffer from chain drift by construction; i.e., they automatically satisfy Walsh's multiperiod identity test.**
- **We prefer the use of maximum overlap indexes rather than using carry forward prices, which can lead to a measurable downward bias in the index if there is general inflation.**

4. The Construction of Annual Indexes using Carry Forward Prices

- **Assuming that each commodity in each season of the year is a separate "annual" commodity is the simplest and theoretically most satisfactory method for dealing with seasonal commodities when the goal is to construct annual price and quantity indexes.**
- **This idea can be traced back to Mudgett in the consumer price context and to Stone in the producer price context.**
- **The *year y annual fixed base Laspeyres price index* using carry forward prices is defined as follows:**

(42) $P_{LFB}^{y} \equiv p^{y}\cdot q^{1}/p^{1}\cdot q^{1}$; $\quad y = 1,\ldots,Y;$

$$= \Sigma_{m=1}^{M}\, p^{y,m}\cdot q^{1,m}/\Sigma_{m=1}^{M}\, p^{1,m}\cdot q^{1,m}$$

$$= \Sigma_{m=1}^{M}\, [p^{y,m}\cdot q^{1,m}/p^{1,m}\cdot q^{1,m}][p^{1,m}\cdot q^{1,m}/\Sigma_{m=1}^{M}\, p^{1,m}\cdot q^{1,m}]$$

$$= \Sigma_{m=1}^{M}\, S_{1,m}\, P_{LFB}^{y,m}$$

4. The Construction of Annual Indexes using Carry Forward Prices: The Laspeyres and Paasche Indexes

- where $S_{1,m} \equiv p^{1,m}\cdot q^{1,m}/\Sigma_{m=1}^{M}\, p^{1,m}\cdot q^{1,m}$ is the month m share of total year 1 expenditure on the seasonal commodities in scope and $P_{LFB}^{y,m} \equiv p^{y,m}\cdot q^{1,m}/p^{1,m}\cdot q^{1,m}$ is the Laspeyres fixed base price index for month m in year y which was defined by (2) in section 2.
- Thus *the annual fixed base Laspeyres price index for year y*, P_{LFB}^{y}, is a year 1 monthly expenditure share weighted *arithmetic* average of the M year over year fixed base Laspeyres monthly indexes for year y.
- Similarly, *the annual fixed base Paasche price index for year y*, P_{PFB}^{y}, is a year y monthly expenditure share weighted *harmonic* average of the M fixed base year over year Paasche monthly indexes for year y.
- Thus the year over year monthly Laspeyres and Paasche indexes can be used to construct their annual counterparts.

The Construction of Annual Mudgett Stone Indexes (cont)

- In the paper, we define annual fixed base and chained Laspeyres, Paasche, Fisher and Törnqvist Theil indexes as well as annual GEKS indexes.
- When taking the geometric average of the Fisher star indexes to form the GEKS indexes, it is not necessary to use bilateral Fisher indexes as the basic bilateral index building blocks; one can use the Törnqvist Theil bilateral formula as the basic bilateral formula.
- The resulting multilateral index is known as the GEKS-T or CCDI index. The table and chart below for annual indexes using carry forward prices also lists this index.
- It turns out that the annual Predicted Share measure of relative price dissimilarity is just the sum (over the 12 months in a year) of the monthly year over year measures of predicted share relative price dissimilarity.

The Construction of Annual Mudgett Stone Indexes (cont)

- The ***real time* set of bilateral links** which minimize the predicted share measures of relative price dissimilarity for the annual data are as follows:

1 – 2 – 3 – 4
         | |
        5 6

Table 9: Annual Predicted Share Measures of Price Dissimilarity Using Carry Forward Prices

	y = 1	y = 2	y = 3	y = 4	y = 5	y = 6
z = 1	0.00000	0.00196	0.00198	0.00176	0.00173	0.00225
z = 2	0.00196	0.00000	0.00107	0.00207	0.00109	0.00264
z = 3	0.00198	0.00107	0.00000	0.00104	0.00068	0.00099
z = 4	0.00176	0.00207	0.00104	0.00000	0.00129	0.00055
z = 5	0.00173	0.00109	0.00068	0.00129	0.00000	0.00102
z = 6	0.00225	0.00264	0.00099	0.00055	0.00102	0.00000

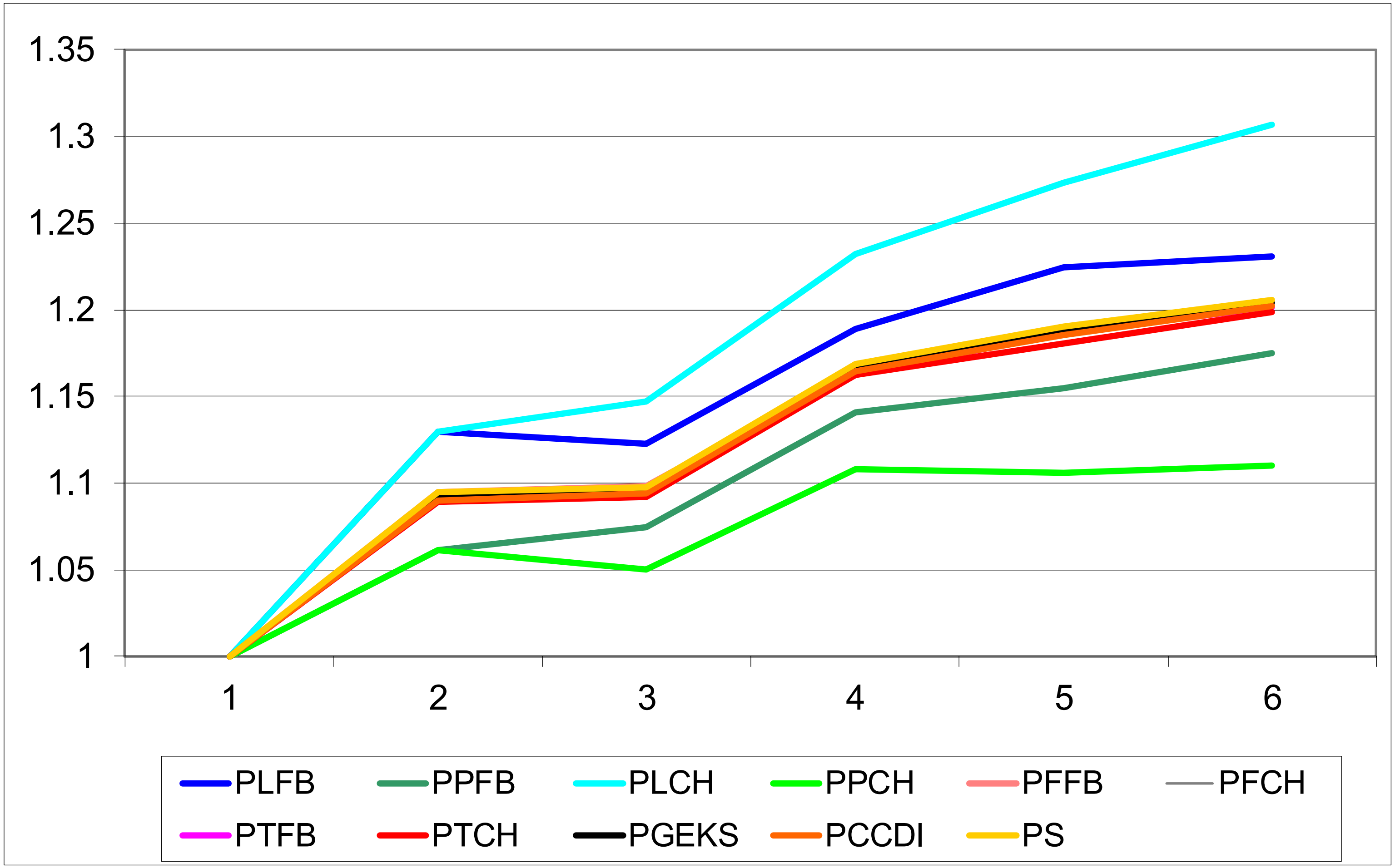
Alternative Annual Mudgett Stone Indexes that Use Year over Year Carry Forward Prices: Table 10

- **The Annual Laspeyres and Paasche indexes are too high or too low while the remaining indexes are fairly close to each other.**

Year	P_{LFB}^y	P_{PFB}^y	P_{LCH}^y	P_{PCH}^y	P_{FFB}^y	P_{FCH}^y	P_{TFB}^y	P_{TCH}^y	P_{GEKS}^y	P_{CCDI}^y	P_S^y
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1299	1.0611	1.1299	1.0611	1.0950	1.0950	1.0892	1.0892	1.0929	1.0900	1.0950
3	1.1224	1.0745	1.1470	1.0502	1.0982	1.0975	1.0963	1.0918	1.0966	1.0941	1.0975
4	1.1891	1.1411	1.2322	1.1083	1.1648	1.1686	1.1624	1.1626	1.1676	1.1647	1.1686
5	1.2241	1.1549	1.2729	1.1060	1.1890	1.1865	1.1871	1.1805	1.1884	1.1856	1.1903
6	1.2306	1.1752	1.3070	1.1102	1.2026	1.2046	1.2015	1.1988	1.2044	1.2020	1.2056
Mean	1.1494	1.1011	1.1815	1.0726	1.1249	1.1254	1.1227	1.1205	1.1250	1.1227	1.1262

Alternative Annual Mudgett Stone Indexes that Use Year over Year Carry Forward Prices: Chart 3

- **The 2 Laspeyres indexes are too high; the 2 Paasche indexes are too low. The other 7 indexes are tightly clustered.**



Alternative Annual Mudgett Stone Indexes that Use Maximum Overlap Bilateral Indexes (No Imputations)

- **Table 11: Imputation Free Annual Index Predicted Share Measures of Price Dissimilarity**
- **The new set of bilateral links in Table 11 is the same as the old set of bilateral links in Table 9.**
- **The magnitude of the measures below is bigger because of the penalty we are now placing on a lack of matching.**

	y = 1	y = 2	y = 3	y = 4	y = 5	y = 6
z = 1	0.00000	0.00284	0.00272	0.00198	0.00272	0.00245
z = 2	0.00284	0.00000	0.00125	0.00275	0.00122	0.00305
z = 3	0.00272	0.00125	0.00000	0.00181	0.00086	0.00148
z = 4	0.00198	0.00275	0.00181	0.00000	0.00213	0.00056
z = 5	0.00272	0.00122	0.00086	0.00213	0.00000	0.00154
z = 6	0.00245	0.00305	0.00148	0.00056	0.00154	0.00000

Alternative Annual Mudgett Stone Indexes that Use Maximum Overlap Bilateral Indexes: Table 12

- **Comparing the results in Tables 10 and 12 shows that the downward bias in the use of carry forward prices in the context of annual indexes is modest; only about 0.16 percentage points per year for our 5 best indexes.**
- **The Charts for the indexes in Tables 10 and 12 are very similar. Table 12 is listed below.**

Year y	P_{LFB}^{y*}	P_{PFB}^{y*}	P_{LCH}^{y*}	P_{PCH}^{y*}	P_{FFB}^{y*}	P_{FCH}^{y*}	P_{TFB}^{y*}	P_{TCH}^{y*}	P_{GEKS}^{y*}	P_{CCDI}^{y*}	P_{S}^{y*}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1373	1.0611	1.1373	1.0611	1.0986	1.0986	1.0921	1.0921	1.0964	1.0930	1.0986
3	1.1273	1.0750	1.1545	1.0468	1.1009	1.0994	1.0986	1.0929	1.0987	1.0958	1.0994
4	1.1919	1.1407	1.2419	1.0985	1.1660	1.1680	1.1633	1.1609	1.1683	1.1650	1.1680
5	1.2253	1.1565	1.2848	1.0962	1.1904	1.1868	1.1876	1.1792	1.1916	1.1881	1.1947
6	1.2344	1.1752	1.3194	1.0973	1.2044	1.2032	1.2031	1.1961	1.2056	1.2028	1.2053
Mean	1.1527	1.1014	1.1897	1.0666	1.1267	1.1260	1.1241	1.1202	1.1268	1.1242	1.1277

Properly Weighted Annual Inflation Measures versus Simple Arithmetic Average of Monthly Measures.

- We conclude this section on annual indexes by looking at some approximations to the "true" Mudgett Stone indexes P_{LFB}^{y*}, P_{PFB}^{y*}, P_{FFB}^{y*}, P_{GEKS}^{y*}, P_{S}^{y*} that are listed in Table 12 (Fixed Base Laspeyres, Paasche, Fisher indexes plus the GEKS and Similarity linked indexes).
- Earlier, we computed the Year over Year monthly Maximum Overlap Fixed Base Laspeyres, Paasche and Fisher indexes, $P_{LFB}^{y,m*}$, $P_{PFB}^{y,m*}$ and $P_{FFB}^{y,m*}$ along with the maximum overlap GEKS index and the Predicted Share Similarity linked indexes, $P_{GEKS}^{y,m*}$ and $P_{S}^{y,m*}$.
- Most statistical agencies compute their annual CPI by taking a simple arithmetic mean of their monthly indexes for the year. But this does not properly weight the contributions of each month, particularly in the seasonal context where monthly expenditures can vary enormously.

Properly Weighted Annual Inflation Measures versus Simple Arithmetic Average of Monthly Measures (cont)

- **Thus define the following *approximate annual indexes* $P_{LFBA}{}^{y*}$, $P_{PFBA}{}^{y*}$, $P_{FFBA}{}^{y*}$, $P_{GEKSA}{}^{y*}$ and $P_{SA}{}^{y*}$ for y = 1,…,Y as follows:**

(80) $P_{LFBA}{}^{y*} \equiv (1/12)\Sigma_{m=1}{}^{12}\, P_{LFB}{}^{y,m*}$;

(81) $P_{PFBA}{}^{y*} \equiv (1/12)\Sigma_{m=1}{}^{12}\, P_{PFB}{}^{y,m*}$;

(82) $P_{FFBA}{}^{y*} \equiv (1/12)\Sigma_{m=1}{}^{12}\, P_{FFB}{}^{y,m*}$;

(83) $P_{GEKSA}{}^{y*} \equiv (1/12)\Sigma_{m=1}{}^{12}\, P_{GEKS}{}^{y,m*}$;

(84) $P_{SA}{}^{y*} \equiv (1/12)\, \Sigma_{m=1}{}^{12}\, P_{S}{}^{y,m*}$.

- **The five "true" annual indexes $P_{LFB}{}^{y*}$, $P_{PFB}{}^{y*}$, $P_{FFB}{}^{y*}$, $P_{GEKS}{}^{y*}$, $P_{S}{}^{y*}$ and their five approximations $P_{LFBA}{}^{y*}$, $P_{PFBA}{}^{y*}$, $P_{FFBA}{}^{y*}$, $P_{GEKSA}{}^{y*}$ and $P_{SA}{}^{y*}$ evaluated using our Israeli data are listed in Table 13 below.**

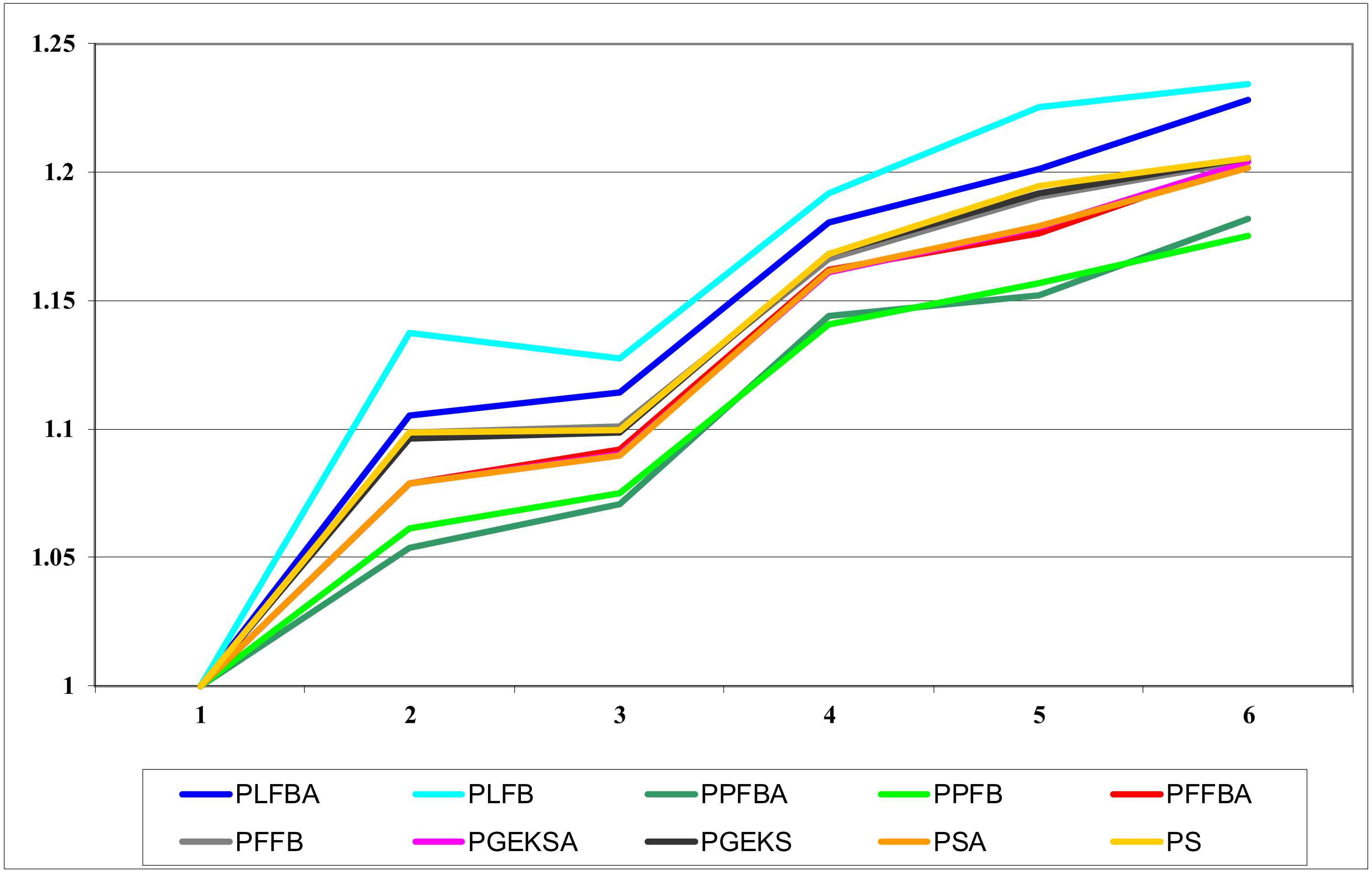
Properly Weighted Annual Inflation Measures versus Simple Arithmetic Average of Monthly Measures (cont)

- **Table 13: Annual Mudgett Stone Indexes Using Maximum Overlap Bilateral Indexes and their Year over Year Simple Approximations:**
- **The next slide plots the 10 indexes listed below.**

Year y	P_{LFBA}^{y*}	P_{LFB}^{y*}	P_{PFBA}^{y*}	P_{PFB}^{y*}	P_{FFBA}^{y*}	P_{FFB}^{y*}	P_{GEKSA}^{y*}	P_{GEKS}^{y*}	P_{SA}^{y*}	P_{S}^{y*}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.1053	1.1373	1.0538	1.0611	1.0789	1.0986	1.0785	1.0964	1.0789	1.0986
3	1.1141	1.1273	1.0706	1.0750	1.0920	1.1009	1.0902	1.0987	1.0896	1.0994
4	1.1802	1.1919	1.1438	1.1407	1.1617	1.1660	1.1612	1.1683	1.1614	1.1680
5	1.2012	1.2253	1.1520	1.1565	1.1761	1.1904	1.1785	1.1916	1.1788	1.1947
6	1.2279	1.2344	1.1817	1.1752	1.2045	1.2044	1.2040	1.2056	1.2014	1.2053
Mean	1.1381	1.1527	1.1003	1.1014	1.1189	1.1267	1.1187	1.1268	1.1184	1.1277

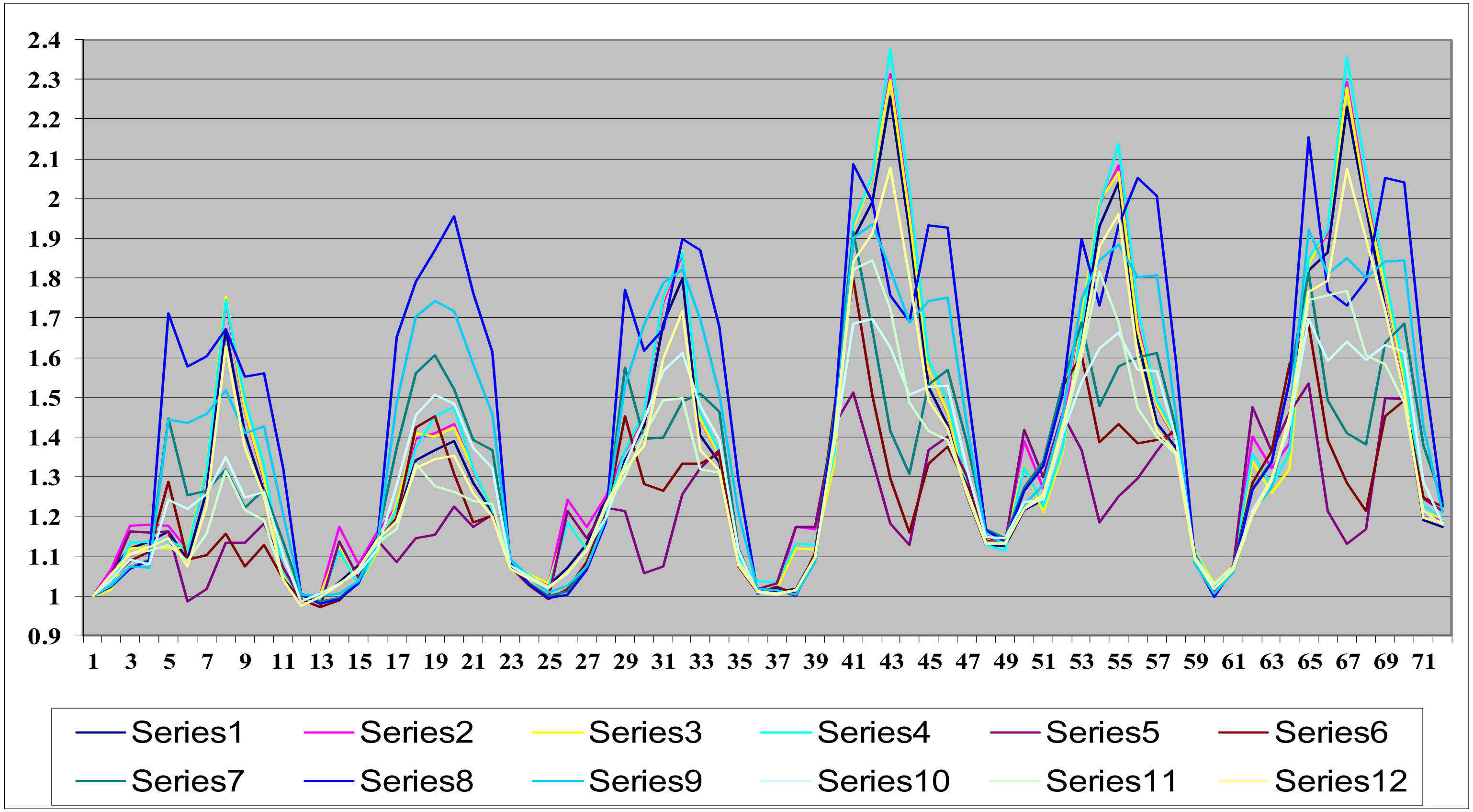
Properly Weighted Annual Inflation Measures versus Simple Arithmetic Average of Monthly Measures (cont)

- **The approximate Laspeyres is not close to its true counterpart. Some of the superlative indexes also differ from their true counterparts. Not a good situation!**



Month to Month Price Indexes: Fisher Star Indexes

- **Below is a chart of the maximum overlap Fixed Base Fisher Star indexes choosing months 1-12 as the base.**
- **The indexes are incredibly volatile and very different as the choice of base month varies.**



Predicted Share Measures of Dissimilarity For Months 1-12

- **The set of real time links which minimize the above dissimilarity measures for the first 12 observations are as follows:**

11

|

1 – 2 – 3 – 4 – 5 – 6 – 7 – 8 – 9 – 10

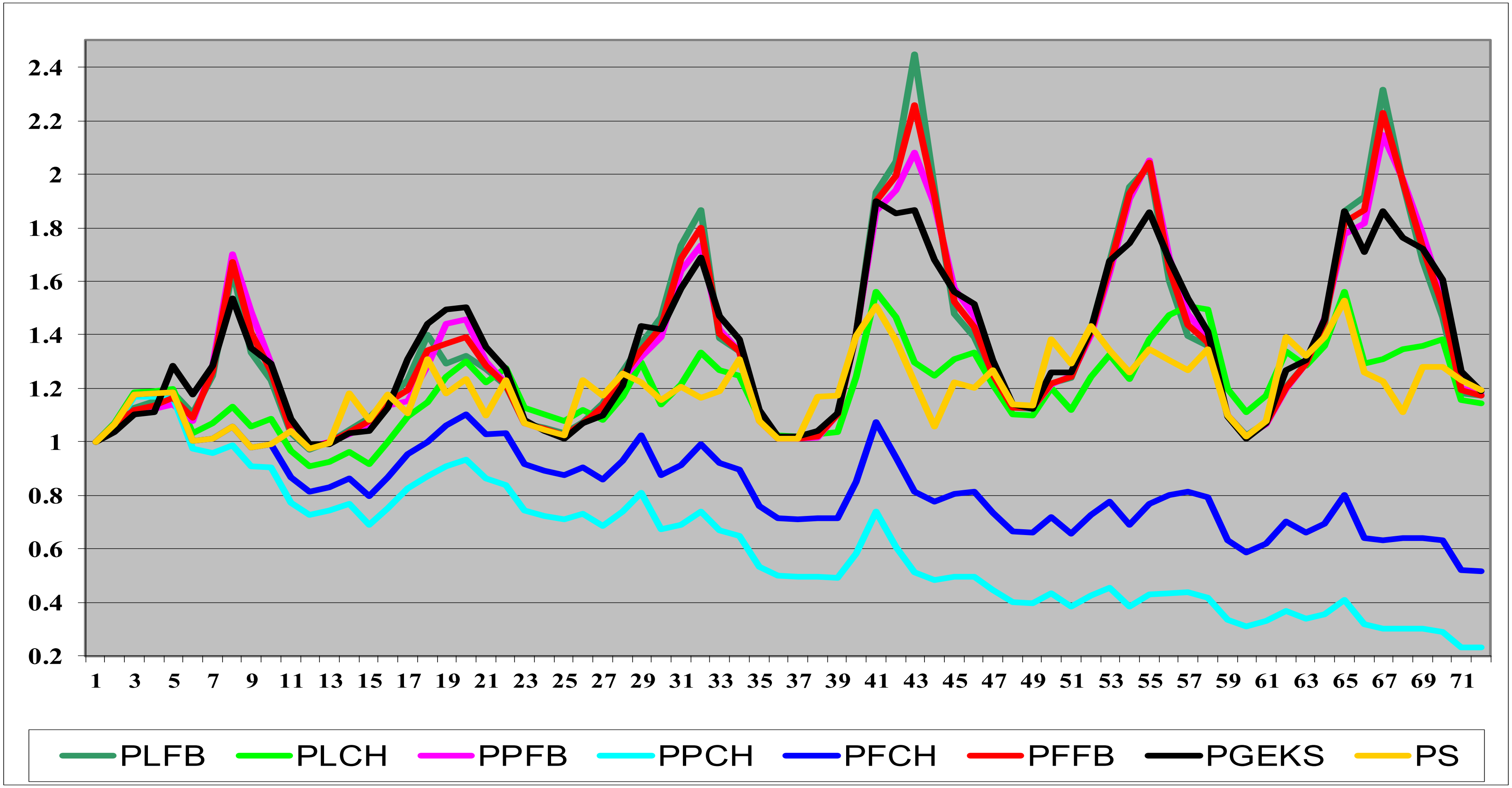
|

12

r,t	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0.1029	0.1075	0.1115	0.4470	0.5477	0.6367	0.6410	0.3713	0.1441	0.0157	0.0022
2	0.1029	0	0.0028	0.0122	0.2387	0.6201	0.6924	0.7014	0.4901	0.2498	0.1198	0.1051
3	0.1075	0.0028	0	0.0062	0.2353	0.6254	0.6967	0.7089	0.4909	0.2562	0.1261	0.1111
4	0.1115	0.0122	0.0062	0	0.2097	0.5398	0.6203	0.6285	0.4593	0.2865	0.1359	0.1073
5	0.4470	0.2387	0.2353	0.2097	0	0.0539	0.0912	0.1017	0.3485	0.2456	0.3900	0.3686
6	0.5477	0.6201	0.6254	0.5398	0.0539	0	0.0250	0.0795	0.2432	0.2248	0.3883	0.4635
7	0.6367	0.6924	0.6967	0.6203	0.0912	0.0250	0	0.0204	0.1716	0.1974	0.3854	0.5560
8	0.6410	0.7014	0.7089	0.6285	0.1017	0.0795	0.0204	0	0.1224	0.1472	0.3619	0.5584
9	0.3713	0.4901	0.4909	0.4593	0.3485	0.2432	0.1716	0.1224	0	0.0148	0.1963	0.3671
10	0.1441	0.2498	0.2562	0.2865	0.2456	0.2248	0.1974	0.1472	0.0148	0	0.0956	0.1429
11	0.0157	0.1198	0.1261	0.1359	0.3900	0.3883	0.3854	0.3619	0.1963	0.0956	0	0.0123
12	0.0022	0.1051	0.1111	0.1073	0.3686	0.4635	0.5560	0.5584	0.3671	0.1429	0.0123	0

Alternative Month to Month Indexes Using Maximum Overlap Bilateral Indexes as Building Blocks: Chart 8

- The chained Paasche and Fisher have large downward chain drifts. The only reasonable indexes are the chained Laspeyres and the Similarity Linked indexes. The seasonal fluctuations in some of the indexes are enormous.



Conclusions on the Month to Month Indexes

- **Chained month to month indexes are too risky; chain drift can be enormous.**
- **Fixed base superlative indexes are also risky; they vary too much as the base changes.**
- **Similarity linked indexes emerge as being the most reasonable. They cannot suffer from chain drift and they had smaller seasonal fluctuations as compared to alternative indexes.**
- **Point of the paper: we need at least 3 separate indexes!**
- **The remainder of the paper considers other indexes:**

  (i) Indexes that use annual baskets or annual shares and

  (ii) Indexes that use only prices.
- **Research problem: how do we integrate a strongly seasonal category with a "regular" month to month CPI?**

Reference: Seasonal Products; Chapter 9 in *CPI Theory*.
https://www.imf.org/en/Data/Statistics/cpi-manual.