Frontier Firms, Inefficiency and Productivity Dynamics

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Outline

Decomposing output growth

- Diewert and Fox (2018) decomposed value added into five explanatory factors:
  - technical progress (τ)
  - inefficiency (ε)
  - input mix (γ)
  - net output prices (α)
  - input quantities (β).
- Advantages of the Diewert and Fox (2018) decomposition:
  - Free Disposal Hull (FDH) and index number theory
  - excludes technical regress
  - a non-parametric approach using only observable data.

Industry-level and firm-level data

Industry	Input	Output
1	…	…
2	…	…
3	…	…

Industry	Firm	Input	Ouput
1	1000001	…	…
1	1000002	…	…
2	1000001	…	…
2	1000002	…	…
3	1000001	…	…
3	1000003	…	…

Marginal revisions

- Frontier firms are confirmed by searching over periods up to and including the current one.
- Output concepts include more than value added.
- Indexes are constructed with a benchmark observation and rolling windows.
- Firms are assumed to share the same price level in one period.

Straightforward decomposition

- Output growth decomposition:

$$\frac{p^t \cdot y_i^t}{p^{t-1} \cdot y_j^{t-1}} = \alpha^t \cdot \beta^t \cdot \gamma^t \cdot \varepsilon^t \cdot \tau^t$$

- TFP growth decomposition:

$$TFPG_{ij}^t = \frac{p^t \cdot y_i^t / p^{t-1} \cdot y_j^{t-1}}{\alpha^t \cdot \beta^t} \\ = \gamma^t \cdot \varepsilon^t \cdot \tau^t$$

Straightforward decomposition

- Fixed base output indexes:

$$\frac{p^t \cdot y_i^t}{p^1 \cdot y_1^1} = A^t \cdot B_i^t \cdot C_i^t \cdot E_i^t \cdot T^t$$

- Fixed base productivity:

$$\begin{aligned} TFP_i^t &= \frac{p^t \cdot y_i^t}{p^1 \cdot y_1^1 \cdot A^t \cdot B_i^t} \\ &= C_i^t \cdot E_i^t \cdot T^t \end{aligned}$$

Firm weighted aggregation

- Firm input quantity weights:

$$w_i^t = \frac{x_i^t}{\sum_{i=1}^{I(t)} x_i^t}$$

- Weighted productivity:

$$\begin{aligned} \ln TFP^t &= \ln \frac{\sum_{i=1}^{I(t)} y_i^t}{\sum_{i=1}^{I(t)} x_i^t} \\ &\approx \sum_{i=1}^{I(t)} w_i^t \ln TFP_i^t \end{aligned}$$

A framework of dynamics

- Decomposing the difference in productivity:

$$\begin{aligned}\Delta \ln TFP^{t,t-1} &= \sum_{i \in U} w_i^t \ln TFP_i^t - \sum_{i \in U} w_i^{t-1} \ln TFP_i^{t-1} \\ &= \sum_{i \in U} w_i^t (\ln C_i^t + \ln E_i^t + \ln T^t) \\ &- \sum_{i \in U} w_i^{t-1} (\ln C_i^{t-1} + \ln E_i^{t-1} + \ln T^{t-1}) \\ &= \Delta \ln C^{t,t-1} + \Delta \ln E^{t,t-1} + \Delta \ln T^{t,t-1}\end{aligned}$$

- Notation: $\Psi \in \{\ln TFP, \ln C, \ln E\}$

BHC decomposition (Baily et al., 1992)

$$
\begin{aligned}
\Delta\Psi^{t,t-1} &= \sum_{i\in U} w_i^t \Psi_i^t - \sum_{i\in U} w_i^{t-1}\Psi_i^{t-1} \\
&= \underbrace{\sum_{i\in U_M} w_i^{t-1}(\Psi_i^t - \Psi_i^{t-1})}_{within} + \underbrace{\sum_{i\in U_M} (w_i^t - w_i^{t-1})\Psi_i^t}_{between} \\
&+ \underbrace{\sum_{i\in U_N} w_i^t \Psi_i^t}_{entry} - \underbrace{\sum_{i\in U_D} w_i^{t-1}\Psi_i^{t-1}}_{exit}
\end{aligned}
$$

GR decomposition (Griliches and Regev, 1995)

$$
\begin{aligned}
\Delta\Psi^{t,t-1} &= \sum_{i\in U} w_i^t(\Psi_i^t - \bar{\Psi}) - \sum_{i\in U} w_i^{t-1}(\Psi_i^{t-1} - \bar{\Psi}) \\
&= \underbrace{\sum_{i\in U_M} \frac{1}{2}(w_i^t + w_i^{t-1})(\Psi_i^t - \Psi_i^{t-1})}_{within} \\
&+ \underbrace{\sum_{i\in U_M} \frac{1}{2}(w_i^t - w_i^{t-1})(\Psi_i^t + \Psi_i^{t-1} - 2\bar{\Psi})}_{between} \\
&+ \underbrace{\sum_{i\in U_N} w_i^t(\Psi_i^t - \bar{\Psi})}_{entry} - \underbrace{\sum_{i\in U_D} w_i^{t-1}(\Psi_i^{t-1} - \bar{\Psi})}_{exit}
\end{aligned}
$$

FHK decomposition (Foster et al., 2001)

$$\Delta\Psi^{t,t-1} = \sum_{i\in U} w_i^t(\Psi_i^t - \Psi^{t-1}) - \sum_{i\in U} w_i^{t-1}(\Psi_i^{t-1} - \Psi^{t-1})$$

$$= \underbrace{\sum_{i\in U_M} w_i^{t-1}(\Psi_i^t - \Psi_i^{t-1})}_{within} + \underbrace{\sum_{i\in U_M} (w_i^t - w_i^{t-1})(\Psi_i^{t-1} - \Psi^{t-1})}_{between}$$

$$+ \underbrace{\sum_{i\in U_M} (w_i^t - w_i^{t-1})(\Psi_i^t - \Psi_i^{t-1})}_{cross}$$

$$+ \underbrace{\sum_{i\in U_N} w_i^t(\Psi_i^t - \Psi^{t-1})}_{entry} - \underbrace{\sum_{i\in U_D} w_i^{t-1}(\Psi_i^{t-1} - \Psi^{t-1})}_{exit}$$

BG decomposition (Baldwin and Gu, 2011)

$$
\begin{aligned}
\Delta\Psi^{t,t-1} &= \sum_{i\in U} w_i^t(\Psi_i^t - \Psi_D) - \sum_{i\in U} w_i^{t-1}(\Psi_i^{t-1} - \Psi_D) \\
&= \underbrace{\sum_{i\in U_M} \frac{1}{2}(w_i^t + w_i^{t-1})(\Psi_i^t - \Psi_i^{t-1})}_{within} \\
&+ \underbrace{\sum_{i\in U_M} \frac{1}{2}(w_i^t - w_i^{t-1})(\Psi_i^t + \Psi_i^{t-1} - 2\Psi_D)}_{between} \\
&+ \underbrace{\sum_{i\in U_N} w_i^t(\Psi_i^t - \Psi_D)}_{entry} - \underbrace{\sum_{i\in U_D} w_i^{t-1}(\Psi_i^{t-1} - \Psi_D)}_{exit}
\end{aligned}
$$

DF decomposition (Diewert and Fox, 2010)

$$\begin{aligned}
\Delta\Psi^{t,t-1} &= \sum_{i\in U} w_i^t \Psi_i^t - \sum_{i\in U} w_i^{t-1}\Psi_i^{t-1} \\
&= \underbrace{\sum_{i\in U_M} \frac{1}{2}(s_{Mi}^t + s_{Mi}^{t-1})(\Psi_i^t - \Psi_i^{t-1})}_{within} \\
&+ \underbrace{\sum_{i\in U_M} \frac{1}{2}(s_{Mi}^t - s_{Mi}^{t-1})(\Psi_i^t + \Psi_i^{t-1})}_{between} \\
&+ \underbrace{s_N^t \sum_{i\in U_N} s_{Ni}^t(\Psi_i^t - \Psi_M^t)}_{entry} - \underbrace{s_D^{t-1} \sum_{i\in U_D} s_{Di}^{t-1}(\Psi_i^{t-1} - \Psi_M^{t-1})}_{exit}
\end{aligned}$$

MP decomposition (Melitz and Polanec, 2015)

$$
\begin{aligned}
\Delta\Psi^{t,t-1} &= \sum_{i\in U} w_i^t \Psi_i^t - \sum_{i\in U} w_i^{t-1}\Psi_i^{t-1} \\
&= \underbrace{\bar{\Psi}_M^t - \bar{\Psi}_M^{t-1}}_{mean} \\
&\quad + \underbrace{\sum_{i\in U_M} \left((s_{Mi}^t - \bar{s}_M^t)(\Psi_i^t - \bar{\Psi}_M^t) - (s_{Mi}^{t-1} - \bar{s}_M^{t-1})(\Psi_i^{t-1} - \bar{\Psi}_M^{t-1}) \right)}_{covariance} \\
&\quad + \underbrace{s_N^t \sum_{i\in U_N} s_{Ni}^t (\Psi_i^t - \Psi_M^t)}_{entry} - \underbrace{s_D^{t-1} \sum_{i\in U_D} s_{Di}^{t-1} (\Psi_i^{t-1} - \Psi_M^{t-1})}_{exit}
\end{aligned}
$$

Revisiting productivity dynamics

- The DID (difference-in-differences) estimator identifies the treatment effect by comparing the treated group and the controlled group.
- Baldwin and Gu (2006) employed a counterfactual specification on entry effect in firm dynamics.
- An causal effect interpretation reveals more components, which is different from Baldwin and Gu (2006).

Real market measures

- Aggregate input shares for group $U_K \in \{U_M, U_D, U_N\}$ are defined as:

$$s_K^{t-1} = \sum_{i \in U_K} w_i^{t-1} / \sum_{i \in U} w_i^{t-1}$$

- Aggregate productivity for group $U_K \in \{U_M, U_D, U_N\}$ are defined as:

$$\Psi_K^{t-1} = \sum_{i \in U_K} s_{Ki}^{t-1} \Psi_i^{t-1}$$

Real market measures

- The industry productivity in period $t-1$ is:

$$\begin{aligned}\Psi^{t-1} &= \sum_{i \in U_M} w_i^{t-1}\Psi_i^{t-1} + \sum_{i \in U_D} w_i^{t-1}\Psi_i^{t-1} \\ &= s_M^{t-1}\Psi_M^{t-1} + s_D^{t-1}\Psi_D^{t-1}\end{aligned}$$

- The industry productivity in period t is:

$$\begin{aligned}\Psi^{t} &= \sum_{i \in U_M} w_i^{t}\Psi_i^{t} + \sum_{i \in U_N} w_i^{t}\Psi_i^{t} \\ &= s_M^{t}\Psi_M^{t} + s_N^{t}\Psi_N^{t}\end{aligned}$$

Pseudo market measures: without entrants

- The industry productivity in period $t-1$ is:

$$\widetilde{\Psi}^{t-1} = s_M^{t-1}\Psi_M^{t-1} + s_D^{t-1}\Psi_D^{t-1}$$

- The industry productivity in period t is:

$$\widetilde{\Psi}^{t} = \Psi_M^{t}$$

- The change of industry productivity without entry effect is:

$$\Delta\widetilde{\Psi}^{t,t-1} = \Psi_M^{t} - s_M^{t-1}\Psi_M^{t-1} - s_D^{t-1}\Psi_D^{t-1}$$

Entry effect

- Comparing the change of industry productivity in the real market and that in the pseudo market:

$$\begin{aligned}\Delta\Psi^{t,t-1} - \Delta\widetilde{\Psi}^{t,t-1} &= s_M^t\Psi_M^t + s_N^t\Psi_N^t - \Psi_M^t \\ &= (1 - s_N^t)\Psi_M^t + s_N^t\Psi_N^t - \Psi_M^t \\ &= s_N^t(\Psi_N^t - \Psi_M^t) \\ &= s_N^t \sum_{i\in U_N} s_{Ni}^t(\Psi_i^t - \Psi_M^t)\end{aligned}$$

- This is exactly the entry effect in DF decomposition.

Pseudo market measures: without exits

- The industry productivity in period $t-1$ is:

$$\widetilde{\Psi}^{t-1} = \Psi_M^{t-1}$$

- The industry productivity in period t is:

$$\widetilde{\Psi}^{t} = s_M^t \Psi_M^t + s_N^t \Psi_N^t$$

- The change of industry productivity without exit effect is:

$$\Delta \widetilde{\Psi}^{t,t-1} = s_M^t \Psi_M^t + s_N^t \Psi_N^t - \Psi_M^{t-1}$$

Exit effect

- Comparing the change of industry productivity in the real market and that in the pseudo market:

$$\begin{aligned}\Delta\Psi^{t,t-1} - \Delta\widetilde{\Psi}^{t,t-1} &= -s_M^{t-1}\Psi_M^{t-1} - s_D^{t-1}\Psi_D^{t-1} + \Psi_M^{t-1} \\ &= -(1 - s_D^{t-1})\Psi_M^{t-1} - s_D^{t-1}\Psi_D^{t-1} + \Psi_M^{t-1} \\ &= -s_D^{t-1}(\Psi_D^{t-1} - \Psi_M^{t-1}) \\ &= -s_D^{t-1}\sum_{i\in U_D} s_{Di}^{t-1}(\Psi_i^{t-1} - \Psi_M^{t-1})\end{aligned}$$

- It is identical to the exit effect in DF decomposition.

Real market measures

- The industry productivity in period $t-1$ is rewritten as:

$$\Psi^{t-1} = \Psi_M^{t-1} + s_D^{t-1}(\Psi_D^{t-1} - \Psi_M^{t-1})$$

- The industry productivity in period t is rewritten as:

$$\Psi^{t} = \Psi_M^{t} + s_N^{t}(\Psi_N^{t} - \Psi_M^{t})$$

- It separates entry effect and exit effect from the effect of incumbents.

Within effect

- The within effect captures the contribution from matched firms of which the firm productivity improves.
- If the firm productivity remains at the level in period $t-1$:

$$\begin{aligned}\Delta\Psi^{t,t-1} - \Delta\widetilde{\Psi}^{t,t-1} &= \Psi_M^t - \widetilde{\Psi}_M^t \\ &= \sum_{i \in U_M} s_{Mi}^t \Psi_i^t - \sum_{i \in U_M} s_{Mi}^t \Psi_i^{t-1} \\ &= \sum_{i \in U_M} s_{Mi}^t (\Psi_i^t - \Psi_i^{t-1})\end{aligned}$$

Within effect

- If the firm productivity remains at the level in period t :

$$\begin{aligned}\Delta\Psi^{t,t-1} - \Delta\widetilde{\Psi}^{t,t-1} &= -\Psi_M^{t-1} + \widetilde{\Psi}_M^{t-1} \\ &= -\sum_{i\in U_M} s_{Mi}^{t-1}\Psi_i^{t-1} + \sum_{i\in U_M} s_{Mi}^{t-1}\Psi_i^{t} \\ &= \sum_{i\in U_M} s_{Mi}^{t-1}(\Psi_i^t - \Psi_i^{t-1})\end{aligned}$$

- An average of the terms in these two scenarios yields:

$$\Delta\Psi^{t,t-1} - \Delta\widetilde{\Psi}^{t,t-1} = \frac{1}{2}\sum_{i\in U_M}(s_{Mi}^t + s_{Mi}^{t-1})(\Psi_i^t - \Psi_i^{t-1})$$

Between effect

- The between effect captures the contribution from matched firms of which the market shares improve.
- If market shares remain at the level in period $t-1$:

$$\begin{aligned}\Delta\Psi^{t,t-1}-\Delta\widetilde{\Psi}^{t,t-1} &= \Psi_M^t-\widetilde{\Psi}_M^t \\ &= \sum_{i\in U_M} s_{Mi}^t\Psi_i^t-\sum_{i\in U_M} s_{Mi}^{t-1}\Psi_i^t \\ &= \sum_{i\in U_M}(s_{Mi}^t-s_{Mi}^{t-1})\Psi_i^t\end{aligned}$$

Between effect

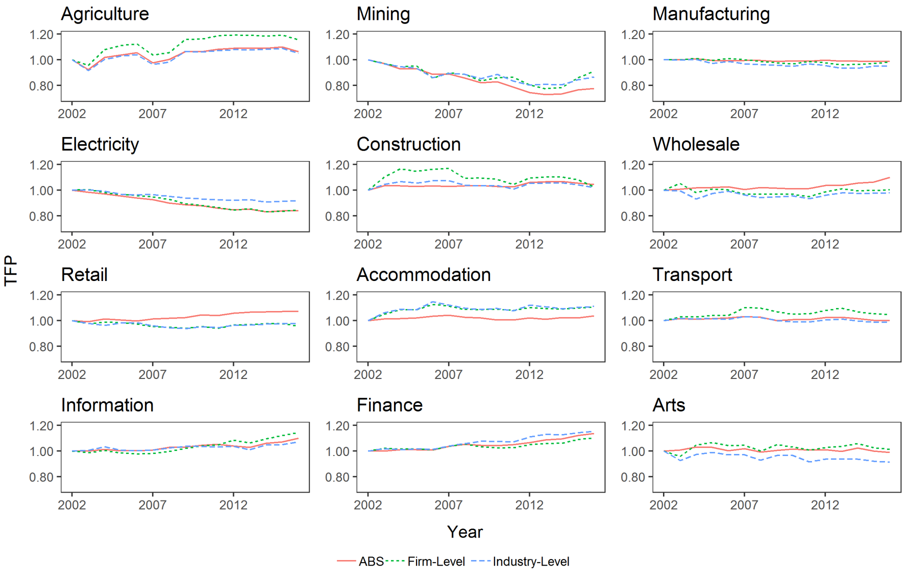
- If market shares remain at the level in period t :

$$\begin{aligned}\Delta\Psi^{t,t-1} - \Delta\widetilde{\Psi}^{t,t-1} &= -\Psi_M^{t-1} + \widetilde{\Psi}_M^{t-1} \\ &= -\sum_{i\in U_M} s_{Mi}^{t-1}\Psi_i^{t-1} + \sum_{i\in U_M} s_{Mi}^{t}\Psi_i^{t-1} \\ &= \sum_{i\in U_M} (s_{Mi}^{t} - s_{Mi}^{t-1})\Psi_i^{t-1}\end{aligned}$$

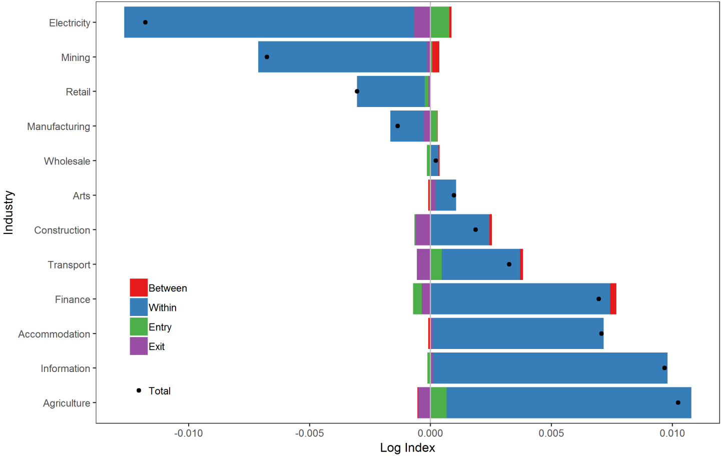
- An average of the terms in these two scenarios yields:

$$\Delta\Psi^{t,t-1} - \Delta\widetilde{\Psi}^{t,t-1} = \frac{1}{2}\sum_{i\in U_M} (s_{Mi}^{t} - s_{Mi}^{t-1})(\Psi_i^{t} + \Psi_i^{t-1})$$

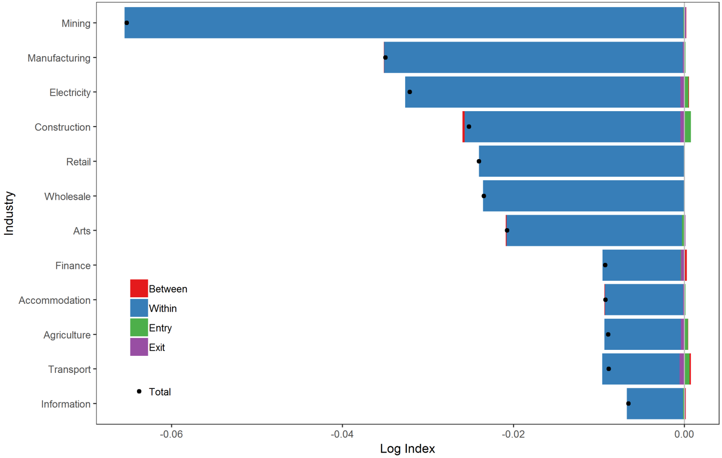
Productivity estimates



Productivity dynamics



Inefficiency dynamics



Conclusion

- A new framework is developed where productivity decomposition is integrated with productivity dynamics.
  - It applies to firm-level data and industry-level data.
  - Explanatory factors of productivity are allowed to be allocated in firm dynamics.
- A DID specification clarifies productivity dynamics within the counterfactual context.
- Empirical evidence from BLADE provides a scan of industry productivity for 12 selective divisions.
  - Disparities between firm-level and industry-level results.
  - Incumbents dominate productivity contribution and efficiency contribution to aggregate performance.

Disclaimer

"The results of these studies are based, in part, on Australian Business Registrar (ABR) data supplied by the Registrar to the ABS under A New Tax System (Australian Business Number) Act 1999 and tax data supplied by the ATO to the ABS under the Taxation Administration Act 1953. These require that such data is only used for the purpose of carrying out functions of the ABS. No individual information collected under the Census and Statistics Act 1905 is provided back to the Registrar or ATO for administrative or regulatory purposes. Any discussion of data limitations or weaknesses is in the context of using the data for statistical purposes, and is not related to the ability of the data to support the ABR or ATO's core operational requirements. Legislative requirements to ensure privacy and secrecy of this data have been followed. Source data are de-identified and so data about specific firms has not been viewed in conducting this analysis. In accordance with the Census and Statistics Act 1905, results have been confidentialised used to ensure that they are not likely to enable identification of a particular person or organisation."

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Defining the optimal output value

- Cost-constrained output value function:

$$R^t(p, w, x) = \max_{y,z}\{p \cdot y : (y, z) \in S^t; w \cdot z \leqslant w \cdot x\}$$

- Unit cost function:

$$c^t(w, p) = \min_{s}\left\{\frac{w \cdot x_s}{p \cdot y_s} : (y_s, x_s) \in S^t\right\}$$

Defining the optimal output value

- Rewrite the cost-constrained output value function:

$$R^t(p, w, x) = \max_s \left\{ p \cdot y_s \frac{w \cdot x}{w \cdot x_s} : (y_s, x_s) \in S^t \right\}$$
$$= \frac{w \cdot x}{c^t(w, p)}$$

- A sequential approach which rules out technical regress.

Explanatory factors

- Net output price indexes:

$$\alpha(p^{t-1}, p^t, w, x, s) = \frac{R^s(p^t, w, x)}{R^s(p^{t-1}, w, x)}$$

- Input quantity indexes:

$$\beta(x_j^{t-1}, x_i^t, w) = \frac{w \cdot x_i^t}{w \cdot x_j^{t-1}}$$

Explanatory factors

- Input mix indexes:

$$\gamma(w^{t-1}, w^t, p, x, s) = \frac{R^s(p, w^t, x)}{R^s(p, w^{t-1}, x)}$$

- Returns to scale:

$$\delta(x_j^{t-1}, x_i^t, p, w, s) = \frac{R^s(p, w, x_i^t)/R^s(p, w, x_j^{t-1})}{w \cdot x_i^t / w \cdot x_j^{t-1}}$$
$$= 1$$

Explanatory factors

- Growth in output efficiency:

$$e_i^t = \frac{p^t \cdot y_i^t}{R^t(p^t, w^t, x_i^t)} \leqslant 1$$

$$e_j^{t-1} = \frac{p^{t-1} \cdot y_j^{t-1}}{R^{t-1}(p^{t-1}, w^{t-1}, x_j^{t-1})} \leqslant 1$$

$$\varepsilon^t = \frac{e_i^t}{e_j^{t-1}}$$

- Technical progress:

$$\tau(t-1, t, p, w, x) = \frac{R^t(p, w, x)}{R^{t-1}(p, w, x)}$$

Rolling windows

- A new set of fixed base output indexes for firms in periods 2, 3, $\cdots$, $N+1$ are computed as:

$$\frac{p^t \cdot y_i^t}{p^2 \cdot y_1^2} = A^{t^*} \cdot B_i^{t^*} \cdot C_i^{t^*} \cdot E_i^{t^*} \cdot T^{t^*}$$

- Linked efficiency indexes for firms in period $N+1$ using mean splices (Diewert and Fox, 2017) are:

$$E_i^{N+1} = E_i^{N+1^*} \cdot \left(\prod_{k,t} \frac{E_k^t}{E_k^{t^*}} \right)^{\frac{1}{M}}$$

Rolling windows

$$
\begin{aligned}
TFP_i^{N+1} &= \frac{p^{N+1} \cdot y_i^{N+1}}{p^1 \cdot y_1^1 \cdot A^{N+1} \cdot B_i^{N+1}} \\
&= \frac{p^{N+1} \cdot y_i^{N+1}}{p^2 \cdot y_1^2 \cdot A^{N+1^*} \cdot B_i^{N+1^*}} \cdot \frac{p^2 \cdot y_1^2}{p^1 \cdot y_1^1 \cdot \left(\prod_{k,t} (A^t \cdot B_k^t)/(A^{t^*} \cdot B_k^{t^*})\right)^{\frac{1}{M}}} \\
&= C_i^{N+1^*} \cdot E_i^{N+1^*} \cdot T^{N+1^*} \cdot \left(\prod_{k,t} \frac{(p^2 \cdot y_1^2 / p^t \cdot y_k^t) \cdot (A^{t^*} \cdot B_k^{t^*})}{(p^1 \cdot y_1^1 / p^t \cdot y_k^t) \cdot (A^t \cdot B_k^t)}\right)^{\frac{1}{M}} \\
&= C_i^{N+1^*} \cdot E_i^{N+1^*} \cdot T^{N+1^*} \cdot \left(\prod_{k,t} \frac{C_k^t \cdot E_k^t \cdot T^t}{C_k^{t^*} \cdot E_k^{t^*} \cdot T^{t^*}}\right)^{\frac{1}{M}} \\
&= C_i^{N+1} \cdot E_i^{N+1} \cdot T^{N+1}
\end{aligned}
$$

GR decomposition

- Griliches and Regev (1995). Firm productivity in Israeli industry 1979–1988. Journal of Econometrics.
- The base period and the current period are involved simultaneously.
- It adopts an average as the benchmark performance:

$$\bar{\Psi} = \frac{1}{2}(\Psi^t + \Psi^{t-1})$$

FHK decomposition

- Foster et al. (2001). Aggregate Productivity Growth: Lessons from Microeconomic Evidence. University of Chicago Press.
- A cross term is newly specified.
- It considers the based period productivity Ψ^{t-1} to be the reference performance.

BG decomposition

- Baldwin and Gu (2011). Plant turnover and productivity growth in Canadian manufacturing. Industrial and Corporate Change.
- The reference productivity is proposed to be the aggregated productivity of disappearing firms Ψ_D.
- New firms enter the industry to replace disappearing firms.

DF decomposition

- Diewert and Fox (2010). On measuring the contribution of entering and exiting firms to aggregate productivity growth. Price and productivity measurement: Volume 6 – index number theory.
- The reference performance is the aggregate productivity of incumbent firms: Ψ_M^t and Ψ_M^{t-1}.

DF decomposition

- Micro input shares for group $U_K \in \{U_M, U_D, U_N\}$ are defined as:

$$s_{Ki}^t = w_i^t / \sum_{i \in U_K} w_i^t$$

- Aggregate input shares for group $U_K \in \{U_M, U_D, U_N\}$ are defined as:

$$s_K^t = \sum_{i \in U_K} w_i^t / \sum_{i \in U} w_i^t$$

MP decomposition

- Melitz and Polanec (2015). Dynamic Olley-Pakes productivity decomposition with entry and exit. The RAND Journal of Economics.
- It shares the same entry effect and exit effect as components in DF decomposition.
- A mean term and a covariance term are added to the incumbents.

MP decomposition

- The unweighted average productivity is:

$$\bar{\Psi}_M^t = \frac{1}{N(M)} \sum_{i \in U_M} \Psi_i^t$$

- The unweighted firm weight is:

$$\begin{aligned} \bar{s}_M^t &= \frac{1}{N(M)} \sum_{i \in U_M} s_{Mi}^t \\ &= \frac{1}{N(M)} \end{aligned}$$