Pondering over the CES Price Index

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“As we disaggregate, we can expect to encounter increasingly large elasticities of substitution.”

(Erwin Diewert, 1974)

EMG Workshop 2005, Tuesday 13 December

9:00 Daniel Melser (Office of National Statistics, UK)

“Package Size and Nonlinear Pricing: Evidence and Implications from Scanner Data”

9:45 Jan de Haan (Statistics Netherlands)

“Quality Change, New and Disappearing Goods and the CES Cost of Living Index”

10:30 Break

11:00 Lorraine Ivancic (UNSW)

“Elasticities of Substitution across Time and Commodity Groups: An Investigation using Scanner Data”

Introduction

Constant Elasticity of Substitution (CES) cost-of-living index or CES price index

Recent research includes Ivancic, Diewert and Fox (2010), Diewert (2018), Diewert and Fox (2018), Melser (2018), Melser and Webster (2019), Redding and Weinstein (2019).

Many recent papers examine the treatment of new and disappearing products, but my presentation is about the CES index for the simple case when the set of products is fixed across time.

Just wanted to get a better understanding; this talk merely reflects my ignorance.

Overview

- Some well-known facts
- A range of representations
- Econometric modelling
- The TPD method and the CES index
- Economic theory and price measurement
- Conclusions

Some well-known facts

CES price index (t=0,…,T)

$$P_{CES}^{0t} = \left[\frac{\sum_{i \in I} b_i (p_i^t)^{1-\sigma}}{\sum_{i \in I} b_i (p_i^0)^{1-\sigma}} \right]^{\frac{1}{1-\sigma}} = \left[\frac{\sum_{i \in I} b_i (p_i^0)^{1-\sigma} \left(\frac{p_i^t}{p_i^0} \right)^{1-\sigma}}{\sum_{i \in I} b_i (p_i^0)^{1-\sigma}} \right]^{\frac{1}{1-\sigma}}$$

Elasticity of substitution (zero or positive)

$$\sigma = -\frac{\partial \ln(x_i^t / x_j^t)}{\partial \ln(p_i^t / p_j^t)}$$

Assumed constant across all pairs of products *i,j* and across time

Some well-known facts (2)

CES price index is transitive

$$P_{CES}^{0t} = \left[\frac{\sum_{i\in I} b_i (p_i^1)^{1-\sigma}}{\sum_{i\in I} b_i (p_i^0)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} \times \left[\frac{\sum_{i\in I} b_i (p_i^2)^{1-\sigma}}{\sum_{i\in I} b_i (p_i^1)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} \times \dots\times \left[\frac{\sum_{i\in I} b_i (p_i^t)^{1-\sigma}}{\sum_{i\in I} b_i (p_i^{t-1})^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}$$

b_i are taste or quality parameters

Shephard's Lemma: "optimal" expenditure shares are equal to

$$s_i^t = \frac{b_i (p_i^t)^{1-\sigma}}{\sum_{i\in I} b_i (p_i^t)^{1-\sigma}}$$

Some well-known facts (3)

Two expressions for the CES index, using the optimal shares for period 0 or period *t*

$$P_{CES}^{0t}=\left[\sum_{i\in I}s_i^0\left(\frac{p_i^t}{p_i^0}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$, known as Lloyd-Moulton price index

$$P_{CES}^{0t}=\left[\frac{\sum_{i\in I}b_i(p_i^t)^{1-\sigma}}{\sum_{i\in I}b_i\left(\frac{p_i^0}{p_i^t}\right)^{1-\sigma}(p_i^t)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}=\left[\sum_{i\in I}s_i^t\left(\frac{p_i^t}{p_i^0}\right)^{-(1-\sigma)}\right]^{\frac{-1}{1-\sigma}}$$

$\sigma=0$ (no substitution): Laspeyres and Paasche price index, respectively

A range of representations

There are infinitely many representations of the CES price index for each value of the elasticity of substitution if assumptions hold exactly; see also Banerjee (1983)

Dividing the optimal share for period *t* by that for period 0 yields

$$\left(\frac{s_i^t}{s_i^0}\right)^{\frac{1}{\sigma-1}} \frac{p_i^t}{p_i^0} = \left[\frac{\sum_{i\in I} b_i (p_i^t)^{1-\sigma}}{\sum_{i\in I} b_i (p_i^0)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} = P_{CES}^{0t}$$

Any weighted or unweighted arithmetic, harmonic or geometric average of the left-hand side across the set of products will be equal to the CES index

weights a_i^t ($\sum_{i\in I} a_i^t = 1$ in every period *t*)

A range of representations (2)

$$P_{CES}^{0t} = \sum_{i\in I} a_i^t \left(\frac{s_i^t}{s_i^0}\right)^{\frac{1}{\sigma-1}} \left(\frac{p_i^t}{p_i^0}\right) = \prod_{i\in I} \left(\frac{s_i^t}{s_i^0}\right)^{\frac{a_i^t}{\sigma-1}} \left(\frac{p_i^t}{p_i^0}\right)^{a_i^t} = \left[\sum_{i\in I} a_i^t \left(\frac{s_i^t}{s_i^0}\right)^{\frac{1}{1-\sigma}} \left(\frac{p_i^t}{p_i^0}\right)^{-1}\right]^{-1}$$

A few examples

1) $\sigma = 0$

$$P_{CES}^{0t} = \sum_{i\in I} s_i^t \left(\frac{s_i^0}{s_i^t}\right)\left(\frac{p_i^t}{p_i^0}\right) = \sum_{i\in I} s_i^0 \left(\frac{p_i^t}{p_i^0}\right) = P_L^{0t}$$

$$P_{CES}^{0t} = \left[\sum_{i\in I} s_i^0 \left(\frac{s_i^0}{s_i^t}\frac{p_i^t}{p_i^0}\right)^{-1}\right]^{-1} = \left[\sum_{i\in I} s_i^t \left(\frac{p_i^t}{p_i^0}\right)^{-1}\right]^{-1} = P_P^{0t}$$

A range of representations (3)

and also (using quantity shares as weights)

$$P_{CES}^{0t} = \sum_{i\in I} \frac{x_i^t}{\sum_{i\in I} x_i^t} \left(\frac{s_i^0}{s_i^t}\right)\left(\frac{p_i^t}{p_i^0}\right) = \sum_{i\in I} \frac{x_i^t}{\sum_{i\in I} x_i^t} \left(\frac{x_i^0 \big/ \sum_{i\in I} p_i^0 x_i^0}{x_i^t \big/ \sum_{i\in I} p_i^t x_i^t}\right) = \frac{\sum_{i\in I} p_i^t x_i^t \big/ \sum_{i\in I} x_i^t}{\sum_{i\in I} p_i^0 x_i^0 \big/ \sum_{i\in I} x_i^0} = P_{UV}^{0t}$$

2) $\sigma = 2$

$$P_{CES}^{0t} = \sum_{i\in I} s_i^0 \left(\frac{s_i^t}{s_i^0}\right)\left(\frac{p_i^t}{p_i^0}\right) = \sum_{i\in I} s_i^t \left(\frac{p_i^t}{p_i^0}\right) = P_{Pal}^{0t}$$

3) $\sigma = 3$

$$P_{CES}^{0t} = \sum_{i\in I} s_i^0 \left(\frac{s_i^t}{s_i^0}\right)^{\frac{1}{2}}\left(\frac{p_i^t}{p_i^0}\right) = \sum_{i\in I} (s_i^0 s_i^t)^{1/2} \left(\frac{p_i^t}{p_i^0}\right)$$

A range of representations (4)

Unweighted geometric average:

$$P_{CES}^{0t} = \left[\prod_{i \in I} \left(\frac{s_i^t}{s_i^0} \right)^{\frac{1}{\sigma-1}} \right]^{\frac{1}{N}} \prod_{i \in I} \left(\frac{p_i^t}{p_i^0} \right)^{\frac{1}{N}} = \left[\frac{\prod_{i \in I} (s_i^t)^{\frac{1}{N}}}{\prod_{i \in I} (s_i^0)^{\frac{1}{N}}} \right]^{\frac{1}{\sigma-1}} P_J^{0t}$$

adjusted Jevons price index

It is unclear whether ratio between square brackets is greater or smaller than 1; this mainly depends on the variance of the expenditure shares in periods 0 and *t*

PS: Sato-Vartia price index is “exact” for every value of the elasticity of substitution

Econometric modelling

Estimating log of expenditure shares

Taking the natural log of the "optimal" (CES) shares and adding error terms gives

$$\ln(s_i^t) = \beta_i + \beta \ln(p_i^t) + \delta^t + \varepsilon_i^t$$

$$\beta_i = \ln(b_i) \quad \beta = 1 - \sigma \quad \delta^t = -\ln\left(\sum\nolimits_{i \in I} b_i (p_i^t)^{1-\sigma}\right)$$

Estimating equation for pooled data, periods 0,…,*T*

$$\ln(s_i^t) = \beta_N + \sum_{i=1}^{N-1} D_i \beta_i + \beta \ln(p_i^t) + \sum_{t=1}^{T} D_i^t \delta^t + \varepsilon_i^t$$

with dummy variables D_i and D_i^t for products and time periods

Econometric modelling (2)

OLS regression (variance of errors assumed constant)

Two estimators of CES index

$$\hat{P}^{0t}_{CES} = \left[\exp(-\hat{\delta}^t)\right]^{\frac{1}{1-\hat{\sigma}}} = \exp\left(-\frac{\hat{\delta}^t}{\hat{\beta}}\right)$$

$$\widetilde{P}^{0t}_{CES} = \left[\frac{\sum_{i\in I}\hat{b}_i (p_i^t)^{1-\hat{\sigma}}}{\sum_{i\in I}\hat{b}_i (p_i^0)^{1-\hat{\sigma}}}\right]^{\frac{1}{1-\hat{\sigma}}} = \left[\frac{\sum_{i\in I}\exp(\hat{\beta}_i)(p_i^t)^{\hat{\beta}}}{\sum_{i\in I}\exp(\hat{\beta}_i)(p_i^0)^{\hat{\beta}}}\right]^{\frac{1}{\hat{\beta}}}$$

$$\widetilde{P}^{0t}_{CES} = \exp\left(-\frac{\hat{\delta}^t}{\hat{\beta}}\right)\left[\frac{\sum_{i\in I}\hat{s}_i^t}{\sum_{i\in I}\hat{s}_i^0}\right]^{\frac{1}{\hat{\beta}}} = \hat{P}^{0t}_{CES}\left[\frac{\sum_{i\in I}\hat{s}_i^t}{\sum_{i\in I}\hat{s}_i^0}\right]^{\frac{1}{\hat{\beta}}}$$

Econometric modelling (3)

Estimating log of prices: "reverse estimating equation"

$$\ln(p_i^t) = \alpha_N + \sum_{i=1}^{N-1} D_i \alpha_i + \alpha \ln(s_i^t) + \sum_{t=1}^{T} D_i^t \delta^{t*} + \xi_i^t$$

- Is the reverse model useful?
- Errors stem from disturbances in shares; biased estimates?
- Is OLS regression appropriate? OLS gives

$$\widehat{P}_{CES}^{0t} = \exp(\hat{\delta}^{t*}) = \prod_{i \in I} \left[\left(\frac{s_i^t}{s_i^0} \right)^{-\hat{\alpha}} \left(\frac{p_i^t}{p_i^0} \right) \right]^{\frac{1}{N}} = \prod_{i \in I} \left[\left(\frac{s_i^t}{s_i^0} \right)^{\frac{1}{N}} \right]^{\frac{1}{\hat{\sigma}-1}} P_J^{0t}$$

The TPD method and the CES index

Reverse model "is intimately related to the multilateral TPD model" (Melser and Webster, 2019)

Estimating equation for TPD (Time Product Dummy) or fixed effects model

$$\ln(p_i^t) = \alpha_N + \sum_{i=1}^{N-1} D_i \alpha_i + \sum_{t=1}^{T} D_i^t \delta^{t*} + \xi_i^t$$

Thus, $\ln(s_i^t)$ is excluded

Omitted variables problem if the aim is to measure the CES index

OLS regression yields

$$\hat{P}_{TPD(OLS)}^{0t} = \prod_{i \in I} \left(\frac{p_i^t}{p_i^t} \right)^{\frac{1}{N}} = P_J^{0t}$$

The TPD method and the CES index (2)

It follows that

$$\hat{P}^{0t}_{TPD(OLS)} = \left[\frac{\prod_{i \in I} (s_i^t)^{\frac{1}{N}}}{\prod_{i \in I} (s_i^0)^{\frac{1}{N}}} \right]^{\frac{1}{1-\hat{\sigma}}} \quad \widehat{P}^{0t}_{CES}$$

Biased estimator CES price index?

My view: the aim of TPD method is not to measure the CES index. Thus, no omitted variables problem.

But the use of OLS is problematic because it produces the (unweighted) Jevons price index – some form of WLS required when estimating the TPD model

Economic theory and price measurement

- Measurement without theory is useless, and economic measurement requires guidance by economic theory,
- Economic theory is about stylized facts. It cannot, and is not mean to, describe a phenomenon in every detail.
- Simplifying assumptions are made, also for mathematical convenience.

- Utility based consumer demand theory: (Konüs) cost-of-living index can be viewed as the underlying concept of the CPI.
- Assumptions about consumer preferences needed to arrive at a measurable target index

Economic theory and price measurement (2)

- Superlative indexes provide “good approximations” to the cost-of-living index for a wide range of preferences (though not CES).
- Axiomatic/test approach also supports superlative indexes, especially the Fisher ideal index. This approach relies on common sense because it just formalizes reasonable axioms or tests an index should satisfy.
- Some of the CES assumptions are quite restrictive:

  - optimal consumer behavior
  - fixed taste/quality parameters – I do not quite understand their role, especially in a matched-model framework
  - elasticity of substitution that is the same for all pairs of products (and fixed across time)

Economic theory and price measurement (3)

What if behavior is not “optimal”?

With promotional sales for storable goods, quantities purchased tend to spike and only return to normal levels after a while. Often causes (downward) drift in chained price indexes, including superlative indexes.

In this case, disturbances in observed expenditure shares will not be random with a constant variance.

Transitivity property of the CES index will be violated – chain drift

Other “non-optimal” behavior ….

Economic theory and price measurement (4)

What is the role of the taste parameters?

Taste parameters define “quality-adjusted prices “

Without explicitly estimating them, the CES-based expenditure shares implied by prices and a certain value of the elasticity of substitution cannot be calculated (?).

For a heterogeneous product category (of broadly similar products), these parameters cannot be the same if they aim to reflect quality differences in terms of, say, product characteristics.

Economic theory and price measurement (5)

The role of the taste parameters ….

Shouldn't there be a relationship between the variability of the quality parameters and the value of the elasticity of substitution?

- no spread in quality parameters means no quality differences; the product category can then be deemed homogenous and the elasticity should be very large
- big quality differences should lead to substantial variability in the parameters and lower substitution possibilities across many pairs of products

Economic theory and price measurement (6)

Constancy of the elasticity of substitution?

Because quality differences exist within a category consisting of broadly similar products, a certain product will be “more similar” to some products than to others.

Thus, the elasticity of substitution should not be constant across different pairs of products and disaggregation according to similarity will increase homogeneity within the sub-categories.

To summarize:

If quality differences exist, the elasticity of substitution is unlikely to be the same for different pairs of products …. it will generally be non-constant.

Conclusions

- Scanner data tells us that standard assumptions in microeconomic theory are sometimes violated.
- The existence of an elasticity of substitution that is constant across different pairs of products is questionable.

This is not to say that the CES approach is not useful:

assuming constancy of the elasticity, estimating its “average value” and then calculating the Lloyd-Moulton index for a sample of broadly similar products would arguably be an improvement over the traditional sample-based Laspeyres or Lowe index.

Conclusions (2)

The Lloyd-Moulton index can also be calculated at higher aggregation levels to account for “upper level substitution”, but ….

…. I haven’t seen any evidence supporting the assumption of the elasticity of substitution being constant across product categories.