Real GDI, Trading Gains, and Productivity

Ulrich Kohli*

University of Geneva

November 11, 2022

Abstract

Real Gross Domestic Income (GDI) is an important macroeconomic concept that has long been and still is largely neglected by economic analysts. This is due in parts to the fact that there is no widely accepted definition of it and, consequently, of its trading-gain component. In this paper we examine a number of competing definitions of real GDI and we argue that the case in favor of using the price of gross domestic final expenditure as the GDI deflator is overwhelming. The recognition of the central role of real GDI also has implications for the measurement of productivity. Moreover, we argue that the line between productivity growth and trading gains is often somewhat blurred, which is a strong argument in support of considering both effects jointly. The paper looks both at the Laspeyres and the Törnqvist aggregation, and it identifies the functional forms of the underlying technology for which these indices are exact. It also shows how the trading gains really consist of the two separate effects, a terms-of-trade effect and a real-exchange-rate-effect; most statistical agencies consider only the first effect, which suggests that their estimates of the trading gains are incomplete and that their measures of real GDI are conceptually flawed. Our approach recognizes the fact that almost all international trade is in middle products, but we show that our results are still valid in a simpler setting where all traded products are final goods, such as in the well-known Heckscher-Ohlin-Samuelson model.

* Professor emeritus, Geneva School of Economics and Management, University of Geneva. Ulrich.Kohli@hotmail.com, Ulrich.Kohli@unige.ch. Paper prepared for presentation at the 22nd EMG Conference, University of New South Wales, Sydney, NSW, November 25, 2022.

Real GDI, Trading Gains, and Productivity

1. Introduction

Maybe the title of this paper should be “The many faces of real GDI”, but let us nonetheless begin our discussion with *nominal* Gross Domestic Income (GDI). The concept of GDI is familiar to most first-year economics students who know that GDI is essentially equal to the country’s wage bill, plus profits, plus indirect taxes minus subsidies. They also know that, by the national-accounts identity, it is theoretically equal to the Gross Domestic Product (GDP), except for measurement errors, the so-called statistical discrepancy. Neglecting both this discrepancy and net indirect taxes, we can write:

(1) $$v_{GDP,t} \equiv v_{C,t} + v_{I,t} + v_{G,t} + v_{X,t} - v_{M,t} = v_{L,t} + v_{K,t} \equiv v_{GDI,t} \ ,$$

where the *v*’s denote current-dollar values; the components on the GDP side are consumption (*C*), investment (*I*), government purchases (*G*), exports (*X*), and imports (*M*); on the GDI side, we find labor services (*L*) and capital services (*K*); *t* denotes the time period. In what follows, we will use $v_{GDP,t}$ and $v_{GDI,t}$ interchangeably.

Furthermore, gross domestic expenditure (GDE) is defined as:

(2) $$v_{GDE,t} \equiv v_{GDI,t} + v_{M,t} - v_{X,t} = v_{C,t} + v_{I,t} + v_{G,t} \ .$$

In this fundamental linkage between production, income, and expenditure, GDI occupies a pivotal role. Yet, it is a rather elusive figure in the true-world jungle of national account statistics. If one searches for GDI on the U.S. Bureau of Economic analysis (BEA) website, one will almost invariably be directed to GDP. There is no dollar figure to be found for GDI. The best one might find is a reference to the gross-domestic-income measure of real GDP. There is no mention of nominal GDI in the very detailed glossary of the International Monetary Fund (IMF) manual either. This obsession with GDP – and total neglect of its equal GDI – seems rather odd. Why prefer GDP, which is a sign of sweat and tears, over GDI, which is a symbol of economic welfare?

Before proceeding, let us note at the outset that there are a couple of important side issues that we do not address in this paper. Thus, one could certainly argue that it would be preferable to look at the net, rather than the gross concepts of domestic income and product. Indeed, NDP, NDI and NDE are better measures of the country’s production, income and spending levels than their gross counterparts.[1] As we have argued elsewhere,[2] one should even go one step further and look at the national, rather than the domestic concepts.[3] The fixation on the domestic concept probably goes back to 1991, when the U.S. government declared that henceforth it would emphasize GDP over GNP, perhaps in reaction to the growing international indebtedness of the United States. Nonetheless, in order not to add all complications at the same time, we will limit our analysis in this paper to the more familiar gross domestic concepts.

While it is difficult to detect any trace of *nominal* GDI in national account statistics, there are a plethora of competing definitions of *real* GDI that one can find, both in the publications of official statistical agencies and in the literature. This may surprise, since nearly all first-year economics students would have no difficulty in defining real income as nominal income deflated by the price of what it will purchase. Nonetheless, here is a summary of what one can come across:

i) Nominal GDI deflated by the implicit GDP price deflator.[4]
ii) Command-basis GDP, often interpreted as real GDI.[5]
iii) Real GDP plus the trading gains;[6] knowing that there are at least ten different ways to calculate the trading gains,[7] this yields a minimum of ten additional measures of real GDI.[8]
iv) Nominal GDI deflated by the price of gross domestic final expenditure.[9]

This list is not exhaustive. As shown below, there is yet another decomposition of nominal GDI changes between price and quantity effects that one might consider.

It is interesting to note that all these definitions, with the exception of the last one, rely on real GDP or its deflator to define real GDI. Note also that ii) is essentially a special case of iii). Using the IMF definition as a starting point, we recently reviewed five different ways to calculate the trading gains,[10] as suggested by the System of National Accounts (SNA) and the European

[1] This has been emphasized by Diewert and Lawrence (2006) among others.
[2] Kohli (2005).
[3] See the recent article of Grimes and Wu (2022) for an approach along these lines. Actually, these authors go even two steps further by adjusting their data to take account of the depletion of natural resources, and by expressing them in a per capita terms, which is more meaningful from a welfare viewpoint.
[4] Bureau of Economic Analysis (2021), Glossary p. 28.
[5] Denison (1981), Reinsdorf (2010), and Bureau of Economic Analysis (2021), Glossary p. 5.
[6] International Monetary Fund (2009b), page 619.
[7] Silver and Mahdavi (1989).
[8] Hall (2011) is quite right in arguing that GDI so defined is not a check on GDP.
[9] Kohli (2004a, 2006a, 2006b, 2007, 2022, 2023), Reinsdorf (2010).
[10] Kohli (2023).

System of Accounts (ESA)[11]. As we will show below, with one exception, they all lead to theoretically inconsistent measures of real GDI. The one exception is if the trading gains are computed by using the price of gross domestic final expenditure as a deflator of the trade account, in which case it becomes equivalent to iv). Furthermore, given that the BEA today computes command-basis GDP by also using the price of gross domestic final expenditure as a deflator of the trade account, ii) also becomes equivalent to iv). Prior to 2010, the BEA used the price of imports, which indeed led to a conceptually flawed measure of real GDI. The case is far from settled, however, since the overwhelming number of national statistical agencies – including the Australian Bureau of Statistics (ABS) – still compute the trading gains by using another price index (mostly the price of imports) as a deflator of the trade account. The only exception we are aware of besides the BEA is Statistics Canada, which adopted what we view as the correct approach in 2008.[12] The Swiss National Bank did so as well starting even somewhat earlier, in 2007,[13] but inexplicably stopped publication of real GDI in 2014.

This paper proceeds as follows. In the next section we examine the relationship between the main national-account aggregates in the Laspeyres case, which is still the quantity index of choice for nearly all national statistical agencies. We show how real GDI and the trading-gain effect can be derived in a consistent way, and how the latter consists of two components, a terms-of-trade effect and a real exchange-rate effect. A formal derivation is given in Appendix A. Section 3 briefly looks at the approach used by most statistical agencies when deriving the trading gains and real GDI, namely by using the price of imports to deflate the trade account; we show how this approach leads to an internal inconsistency and thus must be decisively rejected. Section 4 generalizes our approach by using a superlative index, the Törnqvist implicit quantity index, in lieu of the Laspeyres quantity aggregation. The formal decomposition of the trading-gain index into a terms-of-trade component and a real-exchange-rate component can also be found in Appendix A. Section 5 looks at the relationship between trading gains and measures of productivity. We argue that in some cases real GDI is a more relevant reference than real GDP when deriving measures of productivity, since trading gains are generally acquired during production rather than after. Moreover, the distinction between trading and productivity gains may be somewhat blurred in many cases; this speaks in favor of considering both sources of income growth jointly. A simple example of when it is difficult to dissociate the two is provided in Appendix B.

[11] International Monetary Fund (2009a), European Commission (2013).
[12] Statistics Canada (2016).
[13] Swiss National Bank (2007).

In what follows we will implicitly use the production-theory approach to modeling imports and exports.[14] This approach recognizes the fact that most international trade is in raw materials and intermediate goods, and that even most so-called finished products that are traded are not ready to meet final demand. Thus, these imports must typically still go through a number of costly domestic transformations, such as unloading, insuring, financing, transporting, wholesaling, storing, repackaging, marketing, advertising, distributing, retailing, and so on, during which they get combined with domestic labor and capital services, so that their final price tag typically substantially exceeds the price paid by the importer, the difference being accounted for by domestic value added. The same is true for exports that must go through similar transformations abroad, and thus are not yet ready to meet final demand either. Treating imports and exports as middle products is also consistent with the approach implicit in the SNA that divides final demand into three main components; these can be viewed as *de facto* nontraded goods.[15] Imports and exports are dealt with separately in the SNA. Although many goods absorbed by domestic residents do have an import component, the price of gross domestic final demand typically does not correlate well with the price of imports, or the price of exports for that matter. Nonetheless, we will also briefly examine in Appendix C the case of the Heckscher-Ohlin-Samuelson model that treats imports and exports as end products, and we will show that our approach is fully valid in that case as well.

2. Real National Accounting in the Laspeyres Case

Let output (including imports, which are treated as a negative output) prices and quantities be denoted by p's and q's. Nominal GDP can be expressed as:

$$(3) \qquad v_{GDP,t} \equiv v_{C,t} + v_{I,t} + v_{G,t} + v_{X,t} - v_{M,t} = p_{C,t}q_{C,t} + p_{I,t}q_{I,t} + p_{G,t}q_{G,t} + p_{X,t}q_{X,t} - p_{M,t}q_{M,t} \ .$$

Assuming that all base-period (period 0) prices have been normalized to unity, it is well known that real GDP can be measured by the following Laspeyres quantity index:

$$(4) \qquad q_{GDP,t} \equiv \frac{v_{C,t}}{p_{C,t}} + \frac{v_{I,t}}{p_{I,t}} + \frac{v_{G,t}}{p_{G,t}} + \frac{v_{X,t}}{p_{X,t}} - \frac{v_{M,t}}{p_{M,t}} = q_{C,t} + q_{I,t} + q_{G,t} + q_{X,t} - q_{M,t} \ .$$

The implicit GDP price deflator therefore has the Paasche form:

[14] See Burgess (1974), Kohli (1978, 1983, 1991), Woodland (1982).
[15] The term "middle product" has been coined by Sanyal and Jones (1982).

(5) $$p_{GDP,t} \equiv \frac{v_{GDP,t}}{q_{GDP,t}} = \frac{1}{s_{C,t}\frac{1}{p_{C,t}} + s_{I,t}\frac{1}{p_{I,t}} + s_{G,t}\frac{1}{p_{G,t}} + s_{X,t}\frac{1}{p_{X,t}} - s_{M,t}\frac{1}{p_{M,t}}} ,$$

where the $s_{i,t}$'s (i = C, I, G, X, M) are the GDP shares of its five components.

Similarly real GDE can be measured by the following Laspeyres index:

(6) $$q_{GDE,t} \equiv \frac{v_{C,t}}{p_{C,t}} + \frac{v_{I,t}}{p_{I,t}} + \frac{v_{G,t}}{p_{G,t}} = q_{C,t} + q_{I,t} + q_{G,t} ,$$

with the corresponding implicit price of gross domestic final expenditure being:

(7) $$p_{GDE,t} \equiv \frac{v_{GDE,t}}{q_{GDE,t}} = \frac{1}{\omega_{C,t}\frac{1}{p_{C,t}} + \omega_{I,t}\frac{1}{p_{I,t}} + \omega_{G,t}\frac{1}{p_{G,t}}} ,$$

where the $\omega_{i,t}$'s indicate the expenditure shares of the three domestic expenditure components (note that $\omega_{i,t} = s_{i,t}$, i = C, I, G, if trade is balanced).

Given that domestic income can ultimately only be used to purchase domestic final goods, $p^{-1}_{GDE,t}$ is the obvious indicator of its purchasing power. Real GDI can thus be measured as:

(8) $$q_{GDI,t} \equiv \frac{v_{GDI,t}}{p_{GDE,t}} = \frac{v_{GDE,t}}{p_{GDE,t}} + \frac{v_{X,t} - v_{M,t}}{p_{GDE,t}} = q_{GDE,t} + q_{X,t}\frac{p_{X,t}}{p_{GDE,t}} - q_{M,t}\frac{p_{M,t}}{p_{GDE,t}} .$$

Applying the IMF definition of real GDI – see iii) above – backwards, we can define the trading gains as:

(9) $$g_{TG,t} \equiv q_{GDI,t} - q_{GDP,t} = q_{X,t}\left(\frac{p_{X,t}}{p_{GDE,t}} - 1\right) - q_{M,t}\left(\frac{p_{M,t}}{p_{GDE,t}} - 1\right) = v_{GDI,t}\left(\frac{1}{p_{GDE,t}} - \frac{1}{p_{GDP,t}}\right) .$$

The real trade balance can be denoted for short as:

(10) $$b_{TB,t} \equiv \frac{v_{X,t} - v_{M,t}}{p_{GDE,t}} .$$

The national accounts identity can therefore be expressed in real terms as:

(11) $$q_{GDP,t} + g_{TG,t} = q_{GDI,t} = q_{GDE,t} + b_{TB,t} ,$$

which once again illustrates the central role of real GDI.

Nominal GDI, defined by (1), can also be written as:

(12) $v_{GDI,t} \equiv v_{L,t} + v_{K,t} = w_{L,t}x_{L,t} + w_{K,t}x_{K,t}$,

where the x's and w's are the quantities and prices of the domestic factor services. Assuming that both user costs are normalized to one in the base period, we can measure the aggregate quantity of factor services by the following Laspeyres index of input quantities:

(13) $x_{GDI,t} = x_{L,t} + x_{K,t}$,

and $w_{GDI,t}$, the implicit user cost deflator, then has the Paasche form:

(14) $$w_{GDI,t} \equiv \frac{v_{GDI,t}}{x_{GDI,t}} = \frac{1}{\sigma_{L,t}\dfrac{1}{w_{L,t}} + \sigma_{K,t}\dfrac{1}{w_{K,t}}} ,$$

the $\sigma_{j,t}$'s ($j = L, K$) being the GDI-shares of the two domestic primary inputs.

Note the analogy between (13) and (4), and between (14) and (5). In (4), $q_{GDP,t}$ was interpreted as real GDP, and hence $p_{GDP,t}$ in (5) as the implicit GDP price deflator. By the same reasoning, one could be tempted to define $x_{GDI,t}$ in (13) as real GDI – yet another definition of real GDI – and $w_{GDI,t}$ in (14) as the implicit GDI price deflator. We will not go this far, however. While $x_{GDI,t}$ and $w_{GDI,t}$ are undeniably magnitudes of great interest, one must keep in mind that we have adopted $p_{GDE,t}^{-1}$ as our measure of purchasing power, and hence one must realize that $w_{GDI,t}$ contains a real element if domestic factors become more productive over time. This leads us to define the Laspeyres index of total factor productivity (TFP) as a Solow residual:

(15) $$r_{TFP,t} \equiv q_{GDP,t} - x_{GDI,t} = v_{GDP,t}\left(\frac{1}{p_{GDP,t}} - \frac{1}{w_{GDI,t}}\right) .$$

As shown in Appendix A, it is possible to decompose the trading-gain index into a terms-of-trade component and a real-exchange-rate component. Let us define the terms of trade, h_t, as:

(16) $$h_t \equiv \frac{p_{X,t} - p_{M,t}}{p_{GDE,t}} .$$

Let $p_{T,t}$ be the price of traded goods:

(17) $$p_{T,t} \equiv \frac{1}{2} p_{X,t} + \frac{1}{2} p_{M,t} \ .$$

We can then define the real exchange rate, e_t, as:[16]

(18) $$e_t \equiv \frac{p_{T,t} - p_{GDE,t}}{p_{GDE,t}} \ .$$

The trading-gain index (9) can then be written as:

(19) $$g_{TG,t} = g_{ToT,t} + g_{RER,t} \ ,$$

where

(20) $$g_{ToT,t} \equiv \frac{1}{2}\left(q_{X,t} + q_{M,t}\right) h_t$$

can formally be interpreted as the terms-of-trade effect, and

(21) $$g_{RER,t} \equiv \left(q_{X,t} - q_{M,t}\right) e_t$$

is the real-exchange-rate effect.[17]

In summary, the full decompositions of real GDP, GDI and GDE then are:

(22) $$q_{GDP,t} = x_{GDI,t} + r_{TFP,t}$$

(23) $$q_{GDI,t} = x_{GDI,t} + r_{TFP,t} + g_{ToT,t} + g_{RER,t} = q_{GDP,t} + g_{ToT,t} + g_{RER,t}$$

(24) $$q_{GDE,t} = x_{GDI,t} + r_{TFP,t} + g_{ToT,t} + g_{RER,t} - b_{TB,t} = q_{GDP,t} + g_{ToT,t} + g_{RER,t} - b_{TB,t} = q_{GDI,t} - b_{TB,t} \ .$$

3. Using the Price of Imports as the Deflator of the Trade Account

The SNA does recommend that statistical agencies compute an estimate of the trading gains, but it makes no firm recommendation regarding the choice of the price deflator of the trade balance. It merely suggests the price of imports, the price of exports, an average of the two, or a general

[16] The rather unusual form of h_t and e_t has to to with the linearity of the underlying model for which the Laspeyres aggregation is exact; see Appendix A for more details. The analogy with more conventional definitions of the real exchange rate and of the terms of trade will become clear in Section 4 below.

[17] See Appendix A for a proof.

price index like the consumer price index or the price of gross domestic final expenditure. It turns out that the use of all of these price indices, except for the last one or unless trade happens to be balanced, lead to measures of the trading gains that are incomplete, as well as estimates of real GDI and of its price deflator that are internally inconsistent. In what follows, we will merely examine the use of the price of imports as the deflator of the trade account. This is the approach used by the BEA until 2010, and by most national statistical agencies around the world, including the Australian Bureau of Statistics, even today.[18] The use of any of the following three price indices mentioned above leads to similar a dead end.[19]

The use of the price of imports as the deflator of the trade account leads to the following estimate of real GDI (or of command-basis GDP to use the terminology of the BEA):

$$(25)\qquad q_{GDI,t}^{M} \equiv q_{C,t} + q_{I,t} + q_{G,t} + \frac{v_{X,t} - v_{M,t}}{p_{M,t}} = q_{GDE,t} + q_{X,t}\frac{p_{X,t}}{p_{M,t}} - q_{M,t} \ .$$

Thus, the only difference between real GDI thus defined and real GDP as given by (3) is the use in (25) of the price of imports to deflate nominal exports. The estimate of the trading gains is then as follows:

$$(26)\qquad g_{TG,t}^{M} \equiv q_{GDI,t}^{M} - q_{GDP,t} = q_{X,t}\left(\frac{p_{X,t}}{p_{M,t}} - 1\right) \ .$$

The implicit GDI price deflator then is:

$$(27)\qquad p_{GDI,t}^{M} \equiv \frac{v_{GDI,t}}{q_{GDI,t}^{M}} = \frac{1}{s_{GDE,t}\dfrac{1}{p_{GDE,t}} + (s_{X,t} - s_{M,t})\dfrac{1}{p_{M,t}}} \ ,$$

with $s_{GDE,t} = s_{C,t} + s_{I,t} + s_{G,t} = 1 - (s_{X,t} - s_{M,t})$. This price index is internally inconsistent since it implies that a change in import prices would affect real income *for a given nominal income and a given price of final expenditure*, unless trade happens to be balanced, an event with probability zero, in which case $p_{GDI,t}^{M}$ would become equal to $p_{GDE,t}$. It would be pointless to argue that a decrease in import prices raises real income because imported goods become cheaper: indeed, any impact of a change in import prices on the prices of the final goods is already caught by $p_{GDE,t}$ and hence it would be incorrect to count it twice. Price index $p_{GDI,t}^{M}$ as measured by (27)

[18] See Australian Bureau of Statistics (2021), page 532 ; this method is also supported by Hall (2011).
[19] See Kohli (2023).

must therefore be decisively rejected, and hence $q_{GDI,t}^{M}$, given by (25), must be viewed as conceptually flawed as well.

The use of the import price as the deflator of the trade account when calculating the trading gains is often justified by arguing that an improvement in the terms off trade makes it possible to increase imports for the same amount of exports. This is correct, but one could equally well argue that it would possible to reduce exports for the same amount of imports, or some combination of the two. In fact, simply increasing imports by the full amount of the trading gains would not make much sense since imports, which largely consist of raw materials and of intermediate products, would still have to be combined with domestic labor and capital services (both of which are in fixed supply) to transform them into goods suited to meet final demand. To liberate some of these resources, some reduction in exports and a reorganization of production would be necessary.

The use of the price of imports as the deflator of the trade account leads to an inconsistency in the national accounts. This inconsistency is identified by the following term:

$$(28) \qquad \Delta_t^M \equiv q_{GDI,t} - q_{GDI,t}^M = g_{TG,t} - g_{TG,t}^M = \left(v_{X,t} - v_{M,t}\right)\left(\frac{1}{p_{GDE,t}} - \frac{1}{p_{M,t}}\right) .$$

Δ_t^M is a measure of the inconsistency of the approach of the ABS and others, and it demonstrates that the measure of the trading gains given by (26) is generally incomplete because it ignores the effect of the change in import prices relative to the prices of domestic goods. The inconsistency also appears in plain sight in the formulation of the national accounts identity:

$$(29) \qquad q_{GDE,t} + b_{TB,t} = q_{GDI,t}^M + \Delta_t^M = q_{GDP,t} + g_{TG,t}^M + \Delta_t^M \ .$$

To sum up, the approach used by most national statistical agencies – with the notable exceptions of Statistics Canada and today's BEA – comes up with a GDI deflator that is generally internally inconsistent. The corresponding measure of real GDI must therefore be viewed as conceptually flawed and rejected as well. One way to deal with this inconsistency, short of correcting it, is to simply ignore it, which probably goes a long way in explaining why real GDI even today is still treated as somewhat of an afterthought, without really been integrated in the national accounts framework.

4. The Törnqvist Approach

The Törnqvist approach is actually fairly straightforward, and we will use our results from the previous sections to just outline the main points.

Let $V_{GDP,t,t-1} \equiv v_{GDP,t} / v_{GDP,t-1}$ be the growth factor of nominal GDP (or, equivalently, nominal GDI); we will use the same notation to designate growth factors of other value, quantity, or price variables. Diewert and Morrison (1986) show that it can be expressed as:

$$(30) \qquad V_{GDP,t,t-1} = P_{GDP,t,t-1} \cdot X_{GDE,t,t-1} \cdot R_{TFP,t,t-1}$$

where

$$(31) \qquad P_{GDP,t,t-1} \equiv \left(\frac{p_{X,t}}{p_{X,t-1}} \right)^{\bar{s}_{X,t}} \left(\frac{p_{M,t}}{p_{M,t-1}} \right)^{-\bar{s}_{M,t}} \left(\frac{p_{GDE,t}}{p_{GDE,t-1}} \right)^{\bar{s}_{GDE,t}}$$

is a Törnqvist index of output prices and

$$(32) \qquad X_{GDI,t,t-1} \equiv \left(\frac{x_{L,t}}{x_{L,t-1}} \right)^{\bar{\sigma}_{L,t}} \left(\frac{x_{K,t}}{x_{K,t-1}} \right)^{\bar{\sigma}_{K,t}}$$

is a Törnqvist index of the quantities of the fixed domestic factors; $s_{i,t}$ ($i = GDE, X, M$) and $\sigma_{j,t}$ ($j = K, L$) are again the nominal GDP/GDI shares of output i and input j at time t, respectively, with $s_{X,t} - s_{M,t} + s_{GDE,t} = 1$ and $\sigma_{L,t} + \sigma_{K,t} = 1$; $\bar{s}_{i,t} \equiv \frac{1}{2}(s_{i,t-1} + s_{i,t})$ and $\bar{\sigma}_{j,t} \equiv \frac{1}{2}(\sigma_{j,t-1} + \sigma_{j,t})$ denote the *average* share of output i and input j over consecutive periods. Diewert and Morrison (1986) demonstrate that both of these indices are exact if the underlying GDP function is Translog.

$R_{TFP,t,t-1}$, finally, is a measure of TFP growth and it is obtained as a residual:[20]

$$(33) \qquad R_{TFP,t,t-1} \equiv V_{GDP,t,t-1} \cdot P^{-1}_{GDP,t,t-1} \cdot X^{-1}_{GDI,t,t-1} \ .$$

Considering expression (30), both $X_{GDI,t,t-1}$ and $R_{TFP,t,t-1}$ are real growth factors and their product yields the real-GDP growth factor:[21]

$$(34) \qquad Q_{GDP,t,t-1} \equiv V_{GDP,t,t-1} / P_{GDP,t,t-1} = X_{GDI,t,t-1} \cdot R_{TFP,t,t-1} \ .$$

[20] It is possible to calculate $R_{t,t-1}$ exactly if the parameters of the Translog GDP function are known; see Kohli (1990).

[21] This index of real GDP thus has the implicit Törnqvist form ; see Kohli (2004b).

This expression shows that the two main sources of economic growth are increases in factor endowments, as captured by $X_{GDI,t,t-1}$, and increases in productivity, as measured by $R_{TFP,t,t-1}$.

Next, in analogy to (8), we obtain the real-GDI growth factor:

(35) $$Q_{GDI,t,t-1} \equiv V_{GDI,t,t-1} \big/ P_{GDE,t,t-1} \ .$$

The trading-gain factor can then be obtained in the same vein as in (9):

(36) $$G_{TG,t,t-1} \equiv Q_{GDI,t,t-1} \big/ Q_{GDP,t,t-1} = P_{GDP,t,t-1} / P_{GDE,t,t-1} \ .$$

This shows that the trading gains can be obtained simply by taking the ratio of two price indices widely available in the national account statistics.

We now re-define the terms of trade (h_t) in the traditional way as the ratio of export prices to import prices:

(37) $$h_t \equiv \frac{p_{X,t}}{p_{M,t}} \ .$$

We next define the price of traded goods ($p_{T,t}$) as the geometric mean of the prices of exports and imports:

(38) $$p_{T,t} \equiv p_{X,t}^{1/2} p_{M,t}^{1/2} \ .$$

Finally, we re-define the real exchange rate (e_t) as the price of traded relative to the price of nontraded goods:[22]

(39) $$e_t \equiv \frac{p_{T,t}}{p_{GDE,t}} = \frac{p_{X,t}^{1/2} p_{M,t}^{1/2}}{p_{GDE,t}} \ .$$

An increase in e_t means, ceteris paribus, a real depreciation of the home currency as internationally traded goods become relative more expensive.

The trading-gain factor can be decomposed as follows:

(40) $$G_{TG,t,t-1} \equiv G_{ToT,t,t-1} \cdot G_{RER,t,t-1} \ ,$$

where:

[22] This measure of the real exchange rate is also known in the literature as the Salter (1959) ratio; on this topic, also see Corden (1992).

(41) $$G_{ToT,t,t-1} \equiv \left(\frac{h_t}{h_{t-1}}\right)^{(\bar{s}_{X,t}+\bar{s}_{M,t})/2} = \left(\frac{p_{X,t}}{p_{X,t-1}}\right)^{(\bar{s}_{X,t}+\bar{s}_{M,t})/2} \left(\frac{p_{M,t}}{p_{M,t-1}}\right)^{-(\bar{s}_{X,t}+\bar{s}_{M,t})/2}$$

measures the terms-of-trade effect, and

(42) $$G_{RER,t,t-1} \equiv \left(\frac{e_t}{e_{t-1}}\right)^{\bar{s}_{X,t}-\bar{s}_{M,t}} = \left(\frac{p_{X,t}}{p_{X,t-1}}\right)^{(\bar{s}_{X,t}-\bar{s}_{M,t})/2} \left(\frac{p_{M,t}}{p_{M,t-1}}\right)^{(\bar{s}_{X,t}-\bar{s}_{M,t})/2} \left(\frac{p_{GDE,t}}{p_{GDE,t-1}}\right)^{-(\bar{s}_{X,t}-\bar{s}_{M,t})}$$

is the real-exchange-rate effect. This decomposition is exact if the underlying GDP function has the Translog form:[23] Note that the welfare effect of a real depreciation of the home currency (an increase in e_t) depends on the position of the trade account as export revenues and the cost of imports both increase: the net effect is positive if the country is in a surplus position, negative otherwise.

In summary, the national accounts relationships can be expressed in real terms as:

(43) $$Q_{GDP,t,t-1} = X_{GDI,t,t-1} \cdot R_{TFP,t,t-1}$$

(44) $$Q_{GDI,t,t-1} = X_{GDI,t,t-1} \cdot R_{TFP,t,t-1} \cdot G_{ToT,t,t-1} \cdot G_{RER,t,t-1} = Q_{GDP,t,t-1} \cdot G_{ToT,t,t-1} \cdot G_{RER,t,t-1}$$

(45) $$Q_{GDE,t,t-1} = X_{GDI,t,t-1} \cdot R_{TFP,t,t-1} \cdot G_{TG,t,t-1} \cdot B^{-1}_{TB,t,t-1} = Q_{GDP,t,t-1} \cdot G_{TG,t,t-1} \cdot B^{-1}_{TB,t,t-1} = Q_{GDI,t,t-1} \cdot B^{-1}_{TB,t,t-1} \ ,$$

where

(46) $$B_{TB,t,t-1} \equiv \frac{Q_{GDI,t,t-1}}{Q_{GDE,t-1}} \approx \left(\frac{V_{X,t,t-1}}{P_{GDE,t,t-1}}\right)^{\bar{s}_{X,t}} \left(\frac{V_{M,t,t-1}}{P_{GDE,t,t-1}}\right)^{-\bar{s}_{M,t}} Q_{GDE,t,t-1}^{-(\bar{s}_{X,t}-\bar{s}_{M,t})} = \left(\frac{V_{X,t,t-1}}{V_{GDE,t,t-1}}\right)^{\bar{s}_{X,t}} \left(\frac{V_{M,t,t-1}}{V_{GDE,t,t-1}}\right)^{-\bar{s}_{M,t}}$$

is a measure of the trade-balance effect.[24]

In analogy to (14), we can define the domestic factor user-cost index as:

(47) $$W_{GDI,t,t-1} \equiv V_{GDI,t,t-1} \big/ X_{GDI,t,t-1} \ .$$

In view of (34), it can be seen that (43) and (44) can also be expressed in the dual price space as:

(48) $$P_{GDP,t,t-1} = W_{GDI,t,t-1} \cdot R^{-1}_{TFP,t,t-1}$$

(49) $$P_{GDE,t,t-1} = W_{GDI,t,t-1} \cdot R^{-1}_{TFP,t,t-1} \cdot G^{-1}_{ToT,t,t-1} \cdot G^{-1}_{RER,t,t-1} = P_{GDP,t,t-1} \cdot G^{-1}_{ToT,t,t-1} \cdot G^{-1}_{RER,t,t-1} \ .$$

[23] For a proof, see Kohli (2006a, 2007) and Appendix A.
[24] This is a quadratic approximation in logarithms to the true effect; we have verified using Australian data that the cumulated effect over the period 1970 to 2019 is correct to the fourth decimal point.

5. Trading gains and productivity

Trading gains and productivity advances are of a similar breed since they both lead to increases in real income for given endowments of primary factors. Moreover, trading gains may affect the measurement of productivity, depending on the definition of productivity that is retained.

One favored measure of productivity has already been referred to, namely total factor productivity (TFP) as captured by Törnqvist index $R_{TFP,t,t-1}$. Identifying the trading gains and adding them to real GDP to get real GDI has no impact on the measures of nominal and real GDP. Changes in the prices of exports, imports, and domestic goods are already fully taken into account when computing nominal GDP and its price. Expression (33) remains valid and the measure of TFP is therefore unaffected. For a given change in the endowment of domestic factors as given by $X_{GDI,t,t-1}$, if properly measured, $R_{TFP,t,t-1}$ is fully determined and thus independent of $G_{ToT,t,t-1}$ and $G_{RER,t,t-1}$. The trading gains simply are a benefit in addition to increases in TFP.

More generally, it is noteworthy that if the Törnqvist aggregation is exact for the underlying function, and assuming perfect competition and optimization, a change in any output price, holding technology and factor endowments constant, has no impact on real GDP since it has exactly the same relative effect on nominal GDP and on its price. Put in another way, using a language familiar to trade economists, a change in output (including import) prices will lead to a movement along the production possibilities frontier, but real GDP, adequately measured, is constant along that line.[25] This not to say that, for given factor endowments and a given technology, a change in the terms of trade or the real exchange rate cannot affect TFP. Quite the contrary: a change in h_t or e_t is likely to have an impact on relative factor rental prices and hence on their income shares, thereby potentially affecting the measure of $X_{GDI,t,t-1}$, and, by the same token, the measure of $R_{TFP,t,t-1}$ obtained as a residual, real GDP remaining unchanged. This, however, is a matter of economic analysis, not an accounting issue. At any point in time, for a given set of output prices, factor endowments and technology, the measure of $R_{TFP,t,t-1}$ is independent of whether or not the trading gains have actually been measured and taken into account.

[25] Technically speaking, it will be a surface in a three-dimensional space rather than just a line since we are considering three variable quantities.

We next consider a second measure of productivity: the average productivity of labor, i.e. the real value added per unit of labor. This is the preferred measure of productivity for many commentators and statistical agencies, including the U.S. Bureau of Labor Statistics (BLS).

In fact, we will consider two such measures, one with respect to real GDP and the other with respect to real GDI. We do, however, have a strong preference for the latter given that international trade takes place overwhelmingly in middle products, and thus occurs during the production process rather than afterwards. As such, we view it as problematic to treat trading gains as an afterthought.

The singling out of labor is also somewhat problematic and needs a justification. In fact, there is no reason to impute productivity and trading gains to labor, as opposed to capital, or both. At best, one can view average labor productivity as a convenient shortcut to relate the overall performance of the economy to the work effort: labor is then used as a metric, so to speak. The wide acceptance of this somewhat Marxist concept probably has to do in parts with its early adoption by the Organisation for European Economic Co-operation (OEEC, the ancestor of the OECD) in 1949 under the influence of Jean Fourastié.[26]

Changes in the terms of trade need not be purely exogenous. Better terms of trade can be the result of a research activity (e.g. market prospection) or of a marketing effort. In a globalized world, firms are constantly searching for new suppliers and additional customers abroad. To the extent that significant quantities of domestic labor and capital are diverted from domestic production to such activities, average labor productivity (and TFP) could be underestimated. Improvement in the terms of trade could also reflect a refinement in the quality of exports that is not fully reflected by the export price and quantity indices. This could also lead to an underestimation of real GDP per unit of labor. Taking the trading gains into account might help to correct for these types of biases.

As already stressed, almost all trade takes place during production, rather than after. In our view the “trade technology”, which “transforms” exports into imports, should therefore be treated as an essential element of the country’s all-embracing technology. Whether components are transformed into others through a physical process, a chemical reaction, or trade, at home or abroad, should not really matter much to economists. Because it may be difficult in many situations to clearly label what is capital deepening, what is technological progress, what is human capital enhancement, and what are pure trading gains, the line between these concepts

[26] See Boulat (2006), p.97.

tends to be blurred in an integrated world. Given the risk that as a result of measurement errors one development may be wrongly imputed to one or another growth factor speaks in favor of considering all of them jointly. Moreover, the reason why economists are interested in productivity is ultimately that it is income enhancing, and it therefore makes sense to take account of all sources of gains, whether domestic or foreign. See Appendix B below for a very simple example where the distinction between total factor productivity and trading gains is rather fuzzy.

Nonetheless, as mentioned earlier, we will also consider the average labor productivity relative to GDP in what follows; as we shall see, the difference between this "closed-economy" measure and the "open-economy" measure we favor is fully accounted for by the trading gains.

We thus begin by defining define $a_{GDI,t} \equiv q_{GDI,t} / x_{L,t}$ as real GDI per unit of labor, or, in terms of growth factors:

(50) $$A_{GDI,t,t-1} \equiv Q_{GDI,t,t-1} \cdot X_{L,t,t-1}^{-1} ,$$

with $A_{GDI,t,t-1} \equiv a_{GDI,t} / a_{GDI,t-1}$ and $X_{L,t,t-1} \equiv x_{L,t} / x_{L,t-1}$. It follows from (44) that this can be expressed as:

(51) $$A_{GDI,t,t-1} = G_{TG,t,t-1} \cdot X_{GDI,t,t-1} \cdot R_{TFP,t,t-1} \cdot X_{L,t,t-1}^{-1} .$$

Making use of (32), we find that:

(52) $$X_{GDI,t,t-1} \cdot X_{L,t,t-1}^{-1} = \left(\frac{x_{K,t}}{x_{L,t}} \right)^{\bar{\sigma}_{K,t}} \left(\frac{x_{L,t}}{x_{L,t-1}} \right)^{\bar{\sigma}_{L,t}-1} = \left(\frac{x_{K,t} / x_{L,t}}{x_{K,t-1} / x_{L,t-1}} \right)^{\bar{\sigma}_{K,t}} = \left(\frac{k_t}{k_{t-1}} \right)^{\bar{\sigma}_{K,t}} \equiv K_{t,t-1} ,$$

with $k_t \equiv x_{K,t} / x_{L,t}$ the capital/labor ratio, and $K_{t,t-1}$ the contribution of capital-intensity changes to economic growth. We thus obtain the following complete Törnqvist decomposition of the growth in this "globalized" version of domestic average labor productivity:

(53) $$A_{GDI,t,t-1} = G_{TG,t,t-1} \cdot K_{t,t-1} \cdot R_{TFP,t,t-1} = G_{ToT,t,t-1} \cdot G_{RER,t,t-1} \cdot K_{t,t-1} \cdot R_{TFP,t,t-1} .$$

This decomposition is exact if the underlying real GDI function is indeed Translog. Admittedly the last two components are likely to dominate the terms-of-trade and the real-exchange-rate effects, but the trading gains need nonetheless to be considered to get a complete assessment of the change in average labor productivity in the open economy.

Note that it follows from (43) and (52) that the product of the last two components yields the growth in the average labor productivity defined with respect to real GDP, $A_{GDP,t,t-1}$, or put another way, the average productivity of labor in a closed-economy setting:[27]

(54) $$A_{GDP,t,t-1} \equiv Q_{GDP,t,t-1} \cdot X_{L,t,t-1}^{-1} = X_{GDI,t,t-1} \cdot R_{TFP,t,t-1} \cdot X_{L,t,t-1}^{-1} = K_{t,t-1} \cdot R_{TFP,t,t-1} \ .$$

Thus, the only difference between this measure and the one we recommend is the exclusion here of the trading gains.

Yet another important indicator of productivity is the *marginal* product of labor. As far as workers are concerned, their marginal product is undoubtedly of more interest to them than their average product since the former is directly related to their purchasing power. In the Cobb-Douglas case, the marginal product of labor is proportional to its average product, but this is generally not true in the case of higher-order functional forms such as the Translog. Under perfect competition and optimization, the marginal product of labor can readily be observed as the real wage rate, $u_{L,t} \equiv w_{L,t} / p_{GDE,t}$, i.e. the nominal wage deflated by the price of domestic final goods, the GDI price deflator. Note that the nominal wage is an income concept and it therefore would make little sense to use the price of GDP as given by (31) to deflate nominal wages. Domestic residents buy domestic final goods, they do not purchase imports or exports. Thus, in view of (36), the trading gains are automatically taken into account in the definition of the real wage, and the question of whether or not the trading gains should be included in this indicator of productivity is a non-issue.

Recall now that $\sigma_{L,t} \equiv (x_{L,t} w_{L,t}) / v_{GDI,t} = (x_{L,t} w_{L,t}) / (q_{GDI,t} p_{GDE,t})$; it therefore follows that $u_{L,t} = a_{GDI,t} \sigma_{L,t}$ or, in terms of growth factors:

(55) $$U_{L,t,t-1} = A_{GDI,t,t-1} \cdot \Sigma_{L,t,t-1} \ ,$$

where

(56) $$U_{L,t,t-1} \equiv u_{L,t} / u_{L,t-1} \text{ and}$$

(57) $$\Sigma_{L,t,t-1} \equiv \sigma_{L,t} / \sigma_{L,t-1} \ .$$

Together with (53), this enables us to get a complete decomposition of the growth of the marginal product of labor:

[27] See Kohli (2022).

(58) $U_{L,t,t-1} = \Sigma_{L,t,t-1} \cdot G_{ToT,t,t-1} \cdot G_{RER,t,t-1} \cdot K_{t,t-1} \cdot R_{TFP,t,t-1}$.

This expression is very handy since each one of its terms can be measured with observed data exclusively. It also shows that, although TFP and capital deepening are almost certainly the main drivers of the growth in the marginal productivity of labor, terms-of-trade and real-exchange-rate effects again cannot be ignored for the decomposition to be complete.

Our results are summarized in Table 1 and illustrated graphically by Figure 1 for Switzerland, 1970-2019[28]. Starting at the bottom of the graph, we first show the path of TFP ($R_{TFP,t}$), chained over the entire period.[29] This line is next augmented by the path of the capital-deepening contributing factor (K_t) to obtain the path of average labor productivity in terms of real GDP ($A_{GDP,t}$); next we have then added the contribution of the trading gains to obtain the path of the average labor productivity in terms of real GDI ($A_{GDI,t}$); finally, multiplying by the labor share index ($\Sigma_{L,t}$), we get the path of the real wage rate, interpreted as the marginal product of labor. It is quite clear that the two main engines of growth of the Swiss economy are the increases in TFP and capital deepening. As expected, the contribution of the trading gains is much smaller, although not insignificant. Thus, in the Swiss case, trading gains have contributed close to 0.1% annually to the growth in real wages. In any case, good accounting practices require that this component not be overlooked.

Decomposition (58) is essentially an accounting identity that should hold at any point in time for a given set of output prices, factor endowments, and technology. It is silent, however, as to the economic forces that cause the changes that are being measured. One must recall that all the components of (58) are endogenous to the extent that they all depend on input and output shares. This is of course most obvious for $\Sigma_{L,t,t-1}$, unless the underlying technology is Cobb-Douglas, in which case $\Sigma_{L,t,t-1} = 1$. The question of how the ratio of the marginal to the average product of labor would change as the result of hypothetical changes in the terms of trade, the real exchange rate, relative factor endowments, and technological progress is an empirical issue, which cannot be answered without a detailed knowledge of the form of the underlying technology. One key

[28] This figure is drawn from Kohli (2022).

[29] Formally, $R_{TFP,t} \equiv R_{TFP,t,t-1} \cdot R_{TFP,t-t,t-2} \cdot \ldots \cdot R_{TFP,1,0} \cdot R_{TFP,0}$ with $R_{TFP,0} = 1$, and similarly for the other growth factors.

parameter is the Hicksian elasticity of complementarity between labor and capital (ψ_{KL}).[30] If ψ_{KL} is greater than one, an increase in the capital-labor ratio will lead to an increase in the labor share, thus meaning that an increase in capital intensity will raise the marginal product of labor by more than its average product. On the other hand, if technological change is mostly Harrod neutral (i.e. labor-augmenting), the passage of time will tend to have an offsetting effect by reducing the labor share for $\psi_{KL} > 1$. Furthermore, although trading gains lead to increases in real domestic income, it is not certain that both factors of production will benefit equally, if at all. It might indeed be the case that one of the two factors gets worse off – even though the country as a whole is unambiguously better off – if its own income share decreases sufficiently.[31] The sign and the size of the impact of changes in the terms of trade and the real exchange rate on the marginal product of labor depend on the so-called Stolper-Samuelson elasticities, which, in turn, are functions of the parameters of the underlying technology.[32]

6. Concluding comments

As shown above, trading gains are important not just for the measurement of real GDI and the determination of aggregate demand, but also for some measures of productivity when defined in a broad context. We have argued that both the measurement of the average and of the marginal productivity of labor should take trading gains into account since almost all trade takes place during – rather than after – production. Some domestic labor is involved in almost all transactions with the rest of the world, and international trade is an intimate part of production in a globalized world. The distinction between capital deepening, technological progress, human capital enhancement, the reorganization of production, and trading gains can be blurred. Appendix B illustrated such an ambiguity with the help of a very simple example. Some advances could be wrongly attributed to one growth factor rather than to another. This calls for an all-encompassing approach where all income-augmenting forces are considered jointly. In fact, when it comes to the marginal productivity of labor, defining the real wage in terms of anything but the purchasing power of domestic income would make little sense.

[30] In the two-input case, the Hicksian elasticity of complementarity is the inverse of the Allen-Uzawa elasticity of substitution.
[31] This is of course the rule in the well-known two-sector Heckscher-Ohlin-Samuelson model of international trade as the result of the implicit, restrictive nonjoint-production hypothesis; see Kohli (1991).
[32] See Kohli (2010) for details.

While we view the trade-in-middle-products approach as the most relevant for the analysis of trading gains, we showed in Appendix C with a simple example that this approach is also valid if trade is assumed to take place in finished products. Either way, the appropriate price deflator of nominal GDI is the price of gross domestic final expenditure.

It is disappointing that the IMF, the OECD, EuroStat, and the United Nations, among others, do not have the resolution to make explicit recommendations concerning the appropriate trade-balance deflator, basically leaving member countries in the dark as to what the best practices are. Thus, it is up to them whether they want to use $p_{M,t}$, $p_{X,t}$, $p_{T,t}$, $p_{GDE,t}$, or yet another price index, as a deflator. Moreover, unless trade happens to be balanced, all the so-called measures of the trading gains using a deflator other than $p_{GDE,t}$ are incomplete since they exclude the relative-price effect resulting from a change in the price of the chosen trade-account deflator relative to the price of domestic final goods. This is why additional components such as $G_{RER,t,t-1}$ are needed. Thus, these official measures are misnamed: they should be viewed at best as measures of the terms-of-trade effects, rather than of the full trading gains. Consequently, the corresponding real GDI estimates must be considered as conceptually flawed.

It would appear that most statistical agencies get it backwards. They select a deflator, more or less at random, receiving no strict guidance from the SNA. They then very carefully calculate the (incomplete) trading gain, add it to their estimate of real GDP, and declare it to be real GDI. The implicit GDI deflator is then almost meaningless since it will generally be a function of the prices of imports and/or exports, incorrectly suggesting that a change in the prices of traded goods would change real domestic income for a *given nominal domestic income and a given domestic price level*. Real GDI then becomes some kind of curiosity in the system of national accounts, with no obvious link to the other aggregates. Instead, these agencies and the authors of the SNA should begin by asking themselves what real GDI is supposed to measure. In our view, the obvious answer is the real purchasing power that is available domestically, at price $p_{GDE,t}$. Once that nominal GDP has been deflated by the price of gross domestic expenditure to yield real GDI, it is straightforward to compute the trading gain with the help of (9) in the Laspeyres case or (36) in the Törnqvist case, as the difference between (or ratio of) real GDI and real GDP, or, which amounts to the same, as the difference between (or the ratio of) the corresponding inverted price indices.

Defining real GDI as nominal GDI deflated by the price of gross domestic expenditure implies that the trade balance must be deflated by that price when computing real GDI in the Laspeyres case as shown by (8). The more than six-decade old question as to what price index should be used to deflate the trade balance would then be answered once for all.[33] A trade surplus is deferred absorption; it is therefore should be clear that to express any trade disequilibrium in real terms the nominal trade balance should be deflated by the price of domestic absorption.

The trading gain can then be decomposed into terms-of-trade and real-exchange-rate effects as shown above. This is all so simple that it is hard to understand why real GDI and the trading-gain concepts are not standard elements of the macroeconomic toolbox. One can only hope that in its next revision, due in 2025, the SNA will provide definite guidance as to what the best practice is.

Real GDP is undoubtedly one of the economic variables the most scrutinized and referred to in practice, by economists, policy makers, and the public at large. Yet, what is real GDP meant to measure? Is it input, is it activity, is it output, is it production, is it real value added, is it real income?[34] For an open economy that trades in middle products, our answer would have to be: none of the above. In our opinion, assuming optimization and perfect competition, real GDP can probably be best viewed as being a metric of the country's domestic production possibilities frontier (PPF). Shifts in the PPF can be explained by changes in domestic factor endowments and in TFP.[35] Whether this rather abstract interpretation of the meaning of real GDP justifies its widespread use by economists and non-economists alike remains an open question.[36] Real GDI, on the other hand, should be straightforward for everyone to understand.

Why are the authors of the SNA so reluctant to do the obvious? Why does the IMF define real GDI as real GDP plus the trading gain, however measured, when historically – and logically – the definitional link between these two concepts went in the opposite direction, with the trading gain originally defined in the literature as the difference between real GDP and the measure of real GDI obtained by deflating the two components of the trade account by a common price index?[37] Could it be the refusal of one large member country to give up its hold on its antiquated

[33] See Burge and Geary (1957).
[34] See Kohli (2007) for a more detailed discussion.
[35] Under allocative inefficiency, measured real GDP might correspond to a point below the PPF; an increase in measured real GDP can therefore also signal an increase in allocative efficiency. Thus, if we depart from the assumption of optimization and perfect competition, it is potential, rather than actual, real GDP that should be thought of as the metric of the PPF. Note, however, that even under allocative inefficiency, actual real GDP could be equal to potential real GDP, and yet real GDI and welfare would not be maximised (i.e. the production point could be on the frontier, but at the wrong place).
[36] Of course, to paraphrase the IMF (see footnote 6 above), we could also define real GDP as real GDI minus the trading gains, but this might not be very appealing either.
[37] See Burge and Geary (1957).

and somewhat mercantilist terminology of “command-basis GNP” and its own bizarre definition of real GDI as nominal GDI (i.e. nominal GDP minus the statistical discrepancy) *deflated by the implicit GDP price index*? It would be time to move on and adopt a definition of real GDI that makes sense, indeed that is intuitively obvious, namely the domestic purchasing power of nominal GDI. As suggested above, the fact that real GDI is measured in practice in all kinds of strange and arbitrary ways is probably the main reason why it has never been recognized as the major macroeconomic variable it really is.

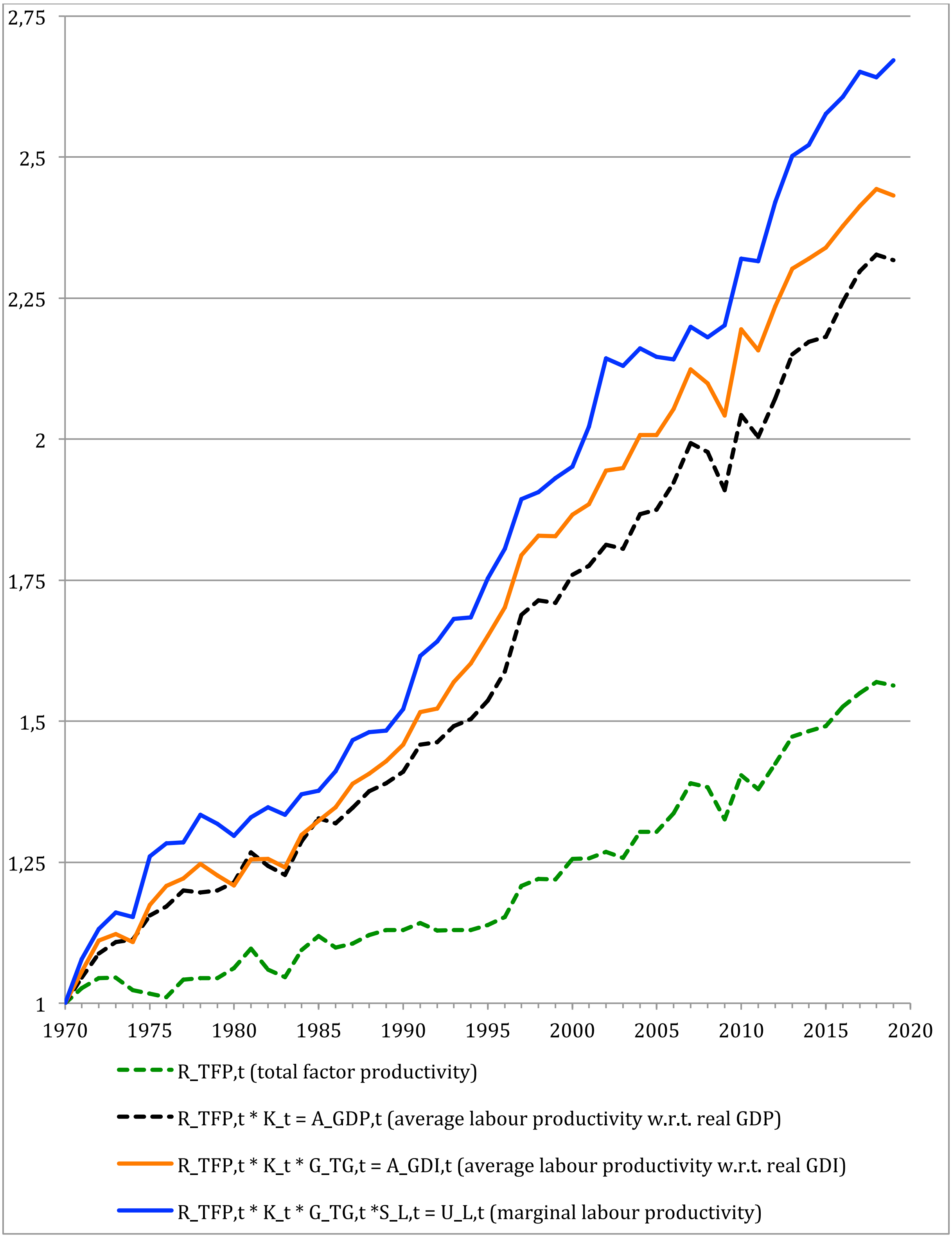
Table 1
Alternative Measures of Productivity-Related Factors
Switzerland, 1970-2019

Year	$R_{TFP,t}$ (33)	K_t (52)	$A_{GDP,t}$ (54)	$G_{TG,t}$ (36)	$A_{GDI,t}$ (50)	$\Sigma_{L,t}$ (57)	$U_{L,t}$ (56)
1970-2019	1.56367	1.48223	2.31771	1.04940	2.43220	1.09866	2.67217
yearly average	1.00916	1.00806	1.01730	1.00098	1.01830	1.00192	1.02026

Figure 1

Decomposition of the marginal productivity of labor

Switzerland, 1970-2019



Appendix A
Derivation of the terms-of-trade and real exchange rate effects in the Laspeyres and the Törnqvist cases

The purpose of this appendix is to give a formal justification for the terms-of-trade and real-exchange-rate gains defined in Sections 2 and 4 of the text.

A.1 The real GDI function

We begin with the definition of real GDI:

$$\text{(A1)} \quad q_{GDI,t} \equiv q_{GDE,t} + q_{X,t} \frac{p_{X,t}}{p_{GDE,t}} - q_{M,t} \frac{p_{M,t}}{p_{GDE,t}} \ .$$

Let the aggregate technology be represented by the following real GDI function:[38]

$$\text{(A2)} \quad \begin{aligned} q_{GDI,t} &= z(p_{GDE,t}, p_{X,t}, p_{M,t}, x_{K,t}, x_{L,t}, t) \\ &\equiv \max_{q_{GDE}, q_X, q_M} \{ q_{GDI} : (q_{GDE}, q_X, q_M, x_{K,t}, x_{L,t}) \in \Psi_t \} \ , \end{aligned}$$

where Ψ_t is the production possibilities set at time *t*. The real GDI function has the following slope properties:[39]

$$\text{(A3)} \quad \frac{\partial z(\cdot)}{\partial p_{GDE}} = \frac{q_{GDE,t}}{p_{GDE,t}} - \frac{q_{GDI,t}}{p_{GDE,t}}$$

$$\text{(A4)} \quad \frac{\partial z(\cdot)}{\partial p_X} = \frac{q_{X,t}}{p_{GDE,t}}$$

$$\text{(A5)} \quad \frac{\partial z(\cdot)}{\partial p_M} = -\frac{q_{M,t}}{p_{GDE,t}} \ .$$

In the Laspeyres case, when definitions (16)–(18) apply, real GDI can be rewritten as:

$$\text{(A6)} \quad q_{GDI,t} = q_{GDE,t} + q_{X,t}(1 + \frac{1}{2} h_t + e_t) - q_{M,t}(1 - \frac{1}{2} h_t + e_t) \ ,$$

whereas in the Törnqvist case, in view of definitions (37)–(39), we have:

[38] Except for the fact that it is deflated by the price of nontraded goods, this is a GDP function like it is well known in the literature; see Diewert (1974), Kohli (1978, 1991), Woodland (1982).
[39] See Kohli (2007).

(A7) $q_{GDI,t} = q_{GDE,t} + q_{X,t} h_t^{1/2} e_t - q_{M,t} h_t^{-1/2} e_t$.

Thus, either way, real GDI can be written as a function of the terms of trade and of the real exchange rate. This means that the aggregate technology can also be represented by the following modified real GDI function:

(A8) $q_{GDI,t} = q_{GDI}(h_t, e_t, x_{K,t}, x_{L,t}, t) \equiv \max_{q_{GDE}, q_X, q_M} \{q_{GDI} : (q_{GDE}, q_X, q_M, x_{K,t}, x_{L,t}) \in \Psi_t\}$.

The passage between formulations (A2) and (A8) is straightforward, once that use of the definitions of h_t and e_t has been made.

A.2 Laspeyres aggregation

Assume that the real GDI function (A2) has the following linear form:

(A9) $q_{GDI,t} = z(p_{GDE,t}, p_{X,t}, p_{M,t}, x_t, \alpha_t) = \left(\alpha_{GDE,t} + \alpha_{X,t} \dfrac{p_{X,t}}{p_{GDE,t}} - \alpha_{M,t} \dfrac{p_{M,t}}{p_{GDE,t}} \right) x_t$,

where $\alpha_{i,t} > 0$ ($i = GDE, X, M$) are technological parameters variable through time as the result of technological change on the variable output side, with $\alpha_t \equiv (\alpha_{GDE,t}, \alpha_{X,t}, \alpha_{M,t})'$ the vector of the $\alpha_{i,t}$'s; $x_t = x(x_{K,t}, x_{L,t}, t)$ is a measure of aggregate domestic input quantities; x_t also registers the effects of technological change on the fixed input side. The Laspeyres aggregation is exact for real GDI function (A9). It follows from (A3)–(A5) that $q_{i,t} = \alpha_{i,t} x_t$, ($i = GDE, X, M$). Real GDP is therefore given by:

(A10) $q_{GDP,t} = (\alpha_{GDE,t} + \alpha_{X,t} - \alpha_{M,t}) x_t = q_{GDE,t} + q_{X,t} - q_{M,t}$.

Making use of (24)–(25), the modified real GDI function (A8) is as follows:

(A11)
$$\begin{aligned} q_{GDI,t} &= q_{GDI}(h_t, e_t, x_t, \alpha_t) = \left[\alpha_{GDE,t} + \alpha_{X,t}(1 + \frac{1}{2} h_t + e_t) - \alpha_{M,t}(1 - \frac{1}{2} h_t + e_t) \right] x_t \\ &= (\alpha_{GDE,t} + \alpha_{X,t} - \alpha_{M,t}) x_t + \frac{1}{2}(\alpha_{X,t} + \alpha_{M,t}) h_t x_t + (\alpha_{X,t} - \alpha_{M,t}) e_t x_t \end{aligned}$$

Remember that $h_0 = e_0 = 0$: this implies that $q_{GDP,t} = q_{GDI}(h_0, e_0, x_t, \alpha_t)$. Consider now the change in real GDI between period 0 and period t. It follows from (A11) that:

$$q_{GDI}(h_t,e_t,x_t,\alpha_t)-q_{GDI}(h_0,e_0,x_0,\alpha_0)=(\alpha_{GDE,t}+\alpha_{X,t}-\alpha_{M,t})x_t-(\alpha_{GDE,0}+\alpha_{X,0}-\alpha_{M,0})x_0$$

$$+\frac{1}{2}(\alpha_{X,t}+\alpha_{M,t})x_t h_t+(\alpha_{X,t}-\alpha_{M,t})x_t e_t$$

$$\text{(A12)}\qquad =q_{GDI}(h_0,e_0,x_t,\alpha_t)-q_{GDI}(h_0,e_0,x_0,\alpha_0)$$

$$+q_{GDI}(h_t,e_t,x_t,\alpha_t)-q_{GDI}(h_0,e_t,x_t,\alpha_t)+q_{GDI}(h_t,e_t,x_t,\alpha_t)-q_{GDI}(h_t,e_0,x_t,\alpha_t)$$

$$=(q_{GDP,t}-q_{GDP,0})+g_{ToT,t}+g_{RER,t}$$

Thus, the change in real GDI between period 0 and period t can be decomposed into three terms: the change in real GDP, the terms-of-trade gain, and the real-exchange-rate gain. The latter two terms were defined in Section 2; see expressions (20) and (21). The terms-of-trade gain component ($g_{ToT,t}$) can be interpreted as the change in real GDI resulting from the change in the terms of trade, holding the real exchange rate and real GDP constant at their period-t levels. The real-exchange-rate gain term ($g_{RER,t}$), on the other hand, can be viewed as the change in real GDI resulting from the change in the real exchange rate, holding the terms of trade and real GDP constant at their period-t levels. These two terms together yield the trading gains, i.e. the difference between real GDI and real GDP.

A.3 Törnqvist aggregation

For the purpose of this section definitions (16)–(18) are replaced by definitions (37)–(39). Consider real GDI function (A8): it can now be rewritten as:

$$\text{(A13)}\quad q_{GDI,t}=q_{GDI}(h_t,e_t,x_{K,t},x_{L,t},t)=q_{GDI}\left(\frac{p_{X,t}}{p_{M,t}},\frac{p_{X,t}^{1/2}p_{M,t}^{-1/2}}{p_{GDE,t}},x_{K,t},x_{L,t},t\right).$$

In view of (A4) and (A5) we find:

$$\text{(A14)}\quad \begin{aligned}\frac{dq_{GDI}(\cdot)}{dp_X}&=\frac{\partial q_{GDI}(\cdot)}{\partial h}\frac{1}{p_{M,t}}+\frac{1}{2}\frac{\partial q_{GDI}(\cdot)}{\partial e}\frac{p_{X,t}^{1/2}p_{M,t}^{-1/2}}{p_{GDE,t}}\\&=\frac{1}{p_{X,t}}\left(h_t\frac{\partial q_{GDI}(\cdot)}{\partial h}+\frac{1}{2}e_t\frac{\partial q_{GDI}(\cdot)}{\partial e}\right)=\frac{q_{X,t}}{p_{GDE,t}}\end{aligned}$$

$$\text{(A15)}\quad \begin{aligned}\frac{dq_{GDI}(\cdot)}{dp_M}&=-\frac{\partial q_{GDI}(\cdot)}{\partial h}\frac{p_{X,t}}{p_{M,t}^2}+\frac{1}{2}\frac{\partial q_{GDI}(\cdot)}{\partial e}\frac{p_{X,t}^{1/2}p_{M,t}^{-1/2}}{p_{GDE,t}}\\&=\frac{1}{p_{M,t}}\left(-h_t\frac{\partial q_{GDI}(\cdot)}{\partial h}+\frac{1}{2}e_t\frac{\partial q_{GDI}(\cdot)}{\partial e}\right)=-\frac{q_{M,t}}{p_{GDE,t}}\end{aligned}$$

Solving (A14)–(A15) for the partial derivatives of $q_{GDI}(\cdot)$ we get:

$$\text{(A16)}\quad \frac{\partial q_{GDI}(\cdot)}{\partial h} = \frac{e_t}{h_t}\left[\frac{1}{2}h_t^{1/2}q_{X,t} + \frac{1}{2}h_t^{-1/2}q_{M,,t}\right]$$

$$\text{(A17)}\quad \frac{\partial q_{GDI}(\cdot)}{\partial e} = h_t^{1/2}q_{X,t} - h_t^{-1/2}q_{M,t} \ .$$

It useful to express these results in elasticity form:

$$\text{(A18)}\quad \frac{\partial q_{GDI}(\cdot)}{\partial h}\frac{h_t}{q_{GDI,t}} = \frac{1}{2}s_{X,t} + \frac{1}{2}s_{M,t}$$

$$\text{(A19)}\quad \frac{\partial q_{GDI}(\cdot)}{\partial e}\frac{e_t}{q_{GDI,t}} = s_{X,t} - s_{M,t} \ .$$

Following along the lines of Diewert and Morrison (1986) we can define the following terms- of-trade effect:

$$\text{(A20)}\quad G_{ToT,t,t-1} \equiv \sqrt{\frac{q_{GDI}(h_t,e_{t-1},x_{K,t-1},x_{L,t-1},t-1)}{q_{GDI}(h_{t-1},e_{t-1},x_{K,t-1},x_{L,t-1},t-1)}\cdot\frac{q_{GDI}(h_t,e_t,x_{K,t},x_{L,t},t)}{q_{GDI}(h_{t-1},e_t,x_{K,t},x_{L,t},t)}} \ .$$

This index, which indicates, *ceteris paribus*, the effect of the change in the terms of trade between time *t-1* and time *t* on real GDI, can be interpreted as the geometric mean of Laspeyres-like and Paasche-like indices, and it thus has the Fisher form so to speak.

We define the real-exchange-rate effect in a similar way:

$$\text{(A21)}\quad G_{RER,t,t-1} \equiv \sqrt{\frac{q_{GDI}(h_{t-1},e_t,x_{K,t-1},x_{L,t-1},t-1)}{q_{GDI}(h_{t-1},e_{t-1},x_{K,t-1},x_{L,t-1},t-1)}\cdot\frac{q_{GDI}(h_t,e_t,x_{K,t},x_{L,t},t)}{q_{GDI}(h_t,e_{t-1},x_{K,t},x_{L,t},t)}} \ .$$

Now assume that the real GDI function (A8) has the Translog form:[40]

$$\text{(A22)}\quad \begin{aligned} \ln q_{GDI,t} &= \alpha_0 + \alpha_H \ln h_t + \alpha_E \ln e_t + \beta_K \ln x_{K,t} + (1-\beta_K)\ln x_{L,t} \\ &+ \frac{1}{2}\gamma_{HH}(\ln h_t)^2 + \gamma_{HE}\ln h_t \ln e_t + \frac{1}{2}\gamma_{EE}(\ln e_t)^2 \\ &+ \frac{1}{2}\varphi_{KK}(\ln x_{K,t} - \ln x_{L,t})^2 + (\delta_{HK}\ln h_t + \delta_{EK}\ln e_t)(\ln x_{K,t} - \ln x_{L,t}) \\ &+ (\delta_{HT}\ln h_t + \delta_{ET}\ln e_t)\,t + \varphi_{KT}(\ln x_{K,t} - \ln x_{L,t})\,t + \beta_T\, t + \frac{1}{2}\varphi_{TT}\, t^2 \end{aligned}$$

[40] See Christensen, Jorgenson, and Lau (1973), Diewert (1974), Kohli (1978, 2006a).

It is well known that the Törnqvist aggregation is exact for the Translog functional form.[41] By introducing (A22) into (A20) and (A21) and making use of (A18)–(A19) one finds that the terms-of-trade and the real-exchange-rate effects as defined above can be measured exactly without knowledge of the parameters of (A22), and that they are precisely equal to expressions (41) and (42) shown in the main text.[42]

[41] See Diewert (1976).
[42] See Kohli (2007) for details.

Appendix B
A simple model of trade in middle products: Trading gains vs. TFP

In this appendix we show with a very simple example how the distinction between TFP and trading gains can be blurred. Assume the following simple economy: labor is the only factor of production; it can be used to produce an intermediate product, which can be exported and traded for an import (e.g. oil), or processed at home, together with labor and imports, to produce a domestic final, nontraded good.

We use the following notation:

$q_{I,t}$: output of intermediate products
$q_{X,t}$: output of exports
$q_{D,t}$: output of intermediate products for domestic use
$q_{M,t}$: purchase of imports
$q_{N,t}$: quantity of domestic final, nontraded good
$x_{LI,t}$: employment in the intermediate good industry
$x_{LN,t}$: employment in the final good industry
$x_{L,t}$: endowment of labor

The corresponding prices are $p_{I,t}$, $p_{X,t}$, $p_{D,t}$, $p_{M,t}$, $p_{N,t}$, and the wage rate is denoted by $w_{L,t}$; note that $p_{X,t} = p_{D,t} = p_{I,t}$ since they relate to the same (middle) product.

Full employment requires that:

(B1) $$x_{L,t} = x_{LI,t} + x_{LN,t} \ ,$$

and equilibrium in the intermediate goods market:

(B2) $$q_{I,t} = q_{X,t} + q_{D,t} \ .$$

The production function for the intermediate product is trivial since labor is the only input:

(B3) $$q_{I,t} = x_{LI,t} \ ,$$

whereas it has the Leontief form for the domestic final good:

(B4) $$q_{N,t} = 3\left[\min\left\{x_{LN,t}, q_{D,t}, q_{M,t}\right\}\right] \ .$$

The corresponding dual cost functions are linear:

(B5) $p_{I,t} = w_{L,t}$

(B6) $p_{N,t} = (w_{L,t} + p_{D,t} + p_{M,t})/3$.

Balanced trade is assumed for simplicity:

(B7) $p_{M,t}q_{M,t} = p_{X,t}q_{X,t}$.

The price of imports is exogenous, and labor is the numeraire, hence we set $w_{L,t}$ = 1. For illustration purposes we set $x_{L,t}$ = 120. Optimizing behaviour is assumed.

Period t = 0: $w_{L,0}$ = 1, $x_{L,0}$ = 120, $p_{M,0}$ = 1

Simple algebra leads to the following solution of the model:

$x_{LI,0} = 80$
$x_{LN,0} = 40$
$q_{I,0} = 80$
$q_{X,0} = q_{D,0} = q_{M,0} = 40$
$q_{N,0} = 120$
$p_{I,0} = p_{X,0} = p_{D,0} = p_{N,0} = 1.$

Furthermore, we find:

Nominal GDP:	$v_{GDP,0} \equiv p_{N,0}q_{N,0} + p_{X,0}q_{X,0} - p_{M,0}q_{M,0} = 120$
Real GDP:	$q_{GDP,0} \equiv q_{N,0} + q_{X,0} - q_{M,0} = 120$
Real GDI:	$q_{GDI,0} \equiv q_{N,0} + (p_{X,0}q_{X,0} - p_{M,0}q_{M,0})/p_{N,0} = v_{GDP,0}/p_{N,0} = 120$
Real GDP per unit of labor:	$a_{GDP,0} \equiv q_{GDP,0}/x_{L,0} = 1$
Real GDI per unit of labor:	$a_{GDI,0} \equiv q_{GDI,0}/x_{L,0} = 1$
Final output per unit of labor:	$q_{N,0}/x_{L,0} = 1$
Real wage:	$u_0 \equiv w_{L,0}/p_{N,0} = 1$.

Period t = 1: $w_{L,1}$ = 1, $x_{L,1}$ = 120, $p_{M,1}$ = ½

The solution then becomes:

$x_{LI,1} = 72$
$x_{LN,1} = 48$

$q_{I,1} = 72$

$q_{X,1} = 24$

$q_{D,1} = q_{M,1} = 48$

$q_{N,1} = 144$

$p_{I,1} = p_{X,1} = p_{D,1} = 1$

$p_{N,1} = 5/6$.

Nominal GDP:	$v_{GDP,1} \equiv p_{N,1}q_{N,1} + p_{X,1}q_{X,1} - p_{M,1}q_{M,1} = 120$
Real GDP:	$q_{GDP,1} \equiv q_{N,1} + q_{X,1} - q_{M,1} = 120$
Real GDI:	$q_{GDI,1} \equiv q_{N,1} + (p_{X,1}q_{X,1} - p_{M,1}q_{M,1}) / p_{N,1} = v_{GDP,1} / p_{N,1} = 144$
Real GDP per unit of labor:	$a_{GDP,,1} \equiv q_{GDP,1} / x_{L,1} = 1$
Real GDI per unit of labor:	$a_{GDI,1} \equiv q_{GDI,1} / x_{L,1} = 1.2$
Final output per unit of labor:	$q_{N,1} / x_{L,1} = 1.2$
Real wage:	$u_1 \equiv w_{L,1} / p_{N,1} = 1.2$
Trading-gain factor:	$G_{TG,1} \equiv q_{GDI,1} / q_{GDP,1} = a_{GDI,1} / a_{GDP,1} = 1.2$.

Thus, in this example, output of the sole domestic final good has unambiguously increased, from 120 to 144, as a result of the improvement in the terms of trade. This gain is not just manna from heaven: some thinking and reorganization is required to take full advantage of it. If we understand labor productivity as pertaining to final output per unit of labor – this is also total factor productivity in this model –, then real GDI per unit of labor appears to be the better "open-economy" measure of labor productivity. Note also that the real wage has gone up in the same proportion as real GDI per unit of labor (the labor share is obviously constant at unity in this model). This simple example shows how the change in the terms of trade can lead to a significant reorganization of production: labor has been shifted from one sector to another, and as a result activity in the intermediate good industry has fallen by 10%, while it has increased by 20% in the final good industry. Real GDP per unit of labor has not gone up, but output per unit of labor certainly has. So is it a gain in productivity, or just a trading gain? The difference between these two concepts seems to be very blurred. This does not matter much, however, as long as the both effects are jointly taken into account.

As mentioned in the main text, real GDP is undoubtedly one of the economic variables the most closely followed by economic actors and observers. Yet, what is real GDP meant to measure?[43] Is it output, is it real value added, is it real income? On the basis of the simple model examined

[43] Swan (2022) addresses the same question.

here, the answer would have to be none of the above. Following the improvement in the terms of trade, output and real value added (i.e. real income) have clearly increased, whereas real GDP has remained unchanged. In this model, one could say that real GDP is equal to activity, i.e. labor input, or, more generally, domestic factor endowments, but this would no longer be true in the presence of technological change. In our opinion, assuming optimization and perfect competition, real GDP can probably be best viewed as being a metric of the country's production possibilities frontier. In our example, the change in the terms of trade has indeed left the country's technology unchanged: it just has made it possible to reach points that were not reachable beforehand.

Appendix C
Traded Goods as Final Goods: Real GDI in the HOS Model

C.1 Theory

While we have emphasized that nearly all trade takes place in middle products, i.e. during production rather than after production, our approach is nonetheless equally relevant for the most popular model of international trade theory, the Heckscher-Ohlin-Samuelson (HOS) model. Note that the HOS model only considers two goods, an importable and an exportable, and thus it does not allow for nontraded goods. The distinction between the terms of trade and the real exchange rate will therefore not be possible. Nonetheless, our simple example will still show that even in this model the case in favor of the use of the price of gross domestic expenditure as the deflator of nominal GDI is overwhelming.

The Heckscher-Ohlin-Samuelson (HOS) model treats trade as taking place after production, and thus involving finished goods exclusively. It is a very elegant model that yields some remarkable theoretical results (e.g. the Stolper-Samuelson Theorem, the Rybczynski Theorem, and the Factor Price Equalization Theorem). However, it also makes some very restrictive assumptions (e.g. nonjoint production), is difficult to generalize, and tricky to implement empirically.[44] Moreover, the assumption that trade happens after production is a drastic departure from reality, given that most trade is raw materials and intermediate products, and that even most so-called finished products that are traded must still go through the domestic production sector before meeting final demand. Furthermore, since trade is viewed as happening after production, one must assume that trade and transportation costs are nil since no real resources are involved. This is a bit odd: if one assums free and instantaneous (indeed, time is costly) transportation, one probably assumes away half of the world economy and infrastructure (the transportation industry; the telecommunication industry; the ship, airline, train, truck and car manufacturers; a good chunk of the energy, construction, tourist, insurance, and banking industries, etc., together with all the harbours, airports, roads, railway tracks and stations, bridges and tunnels, and so on). Much of the climate-change problem could then be assumed away as well…

Note that *de facto* the SNA also depart from the HOS model in a very significant way: by disaggregating final domestic demand into three major components (private consumption, government consumption, and investment), the national accounts implicitly adopt the view that

[44] Kohli (1991).

goods absorbed domestically are distinct from imports and exports. Indeed, the GDE price index typically is only poorly correlated with the prices of imports and exports.

Nonetheless, let us proceed with our look at the HOS model. For illustrative purposes and without any loss of generality, we will assume that the country exports the first good and imports the second one. We will use the following notation:

$y_{i,t}$: output of good i (i = 1,2) at time t

$c_{i,t}$: absorption of good i (i = 1,2) at time t

$p_{i,t}$: price of good i (i = 1,2) at time t

$x_{1,t}$: exports of good 1 at time t

$m_{2,t}$: imports of good 2 at time t

$v_{GDP,t}$: nominal GDP at time t

$v_{GDE,t}$: nominal GDE at time t

$v_{GDI,t}$: nominal GDI at time t

$q_{GDP,t}$: real GDP at time t

$q_{GDE,t}$: real GDE at time t

$q_{GDI,t}$: real GDI at time t

Further variables will be defined as we go along.

Exports are equal to the difference between the production and the absorption of good 1:

(C1) $x_{1,t} = y_{1,t} - c_{1,t}$.

Similarly, imports are given by the excess demand for good 2:

(C2) $m_{2,t} = c_{2,t} - y_{2,t}$.

Nominal GDP can be defined by the demand side or, equivalently, by the output side:

(C3) $v_{GDP,t} = p_{1,t}c_{1,t} + p_{2,t}c_{2,t} + p_{1,t}x_{1,t} - p_{2,t}m_{2,t} = p_{1,t}y_{1,t} + p_{2,t}y_{2,t}$.

By the national accounts identity, nominal GDI is equal to nominal GDP:

(C4) $v_{GDI,t} \equiv v_{GDP,t}$.

Nominal GDE is equal to:

(C5) $v_{GDE,t} = p_{1,t}c_{1,t} + p_{2,t}c_{2,t}$.

Note that if trade were balanced, we would have:

(C6) $$p_{1,t}x_{1,t} = p_{2,t}m_{2,t} \ ,$$

in which case nominal GDE would also be equal to nominal GDP and GDI.

Assuming that all base-period prices are normalized to one, real GDP can be measured by the following Laspeyres quantity index:

(C7) $$q_{GDP,t} = c_{1,t} + c_{2,t} + x_{1,t} - m_{2,t} = y_{1,t} + y_{2,t} \ .$$

The implicit GDP price deflator can then be defined as:

(C8) $$p_{GDP,t} \equiv \frac{v_{GDP,t}}{q_{GDP,t}} = \frac{1}{s_{1,t}\dfrac{1}{p_{1,t}} + s_{2,t}\dfrac{1}{p_{2,t}}} \ ,$$

where $s_{i,t}$ is the share of output *i* in nominal GDP:

(C9) $$s_{i,t} \equiv \frac{p_{i,t}y_{i,t}}{v_{GDP,t}} \quad i = 1,2 \ .$$

As for real GDE, it can also be measured by a Laspeyres quantity index:

(C10) $$q_{GDE,t} = c_{1,t} + c_{2,t} \ ,$$

with the corresponding implicit price deflator:

(C11) $$p_{GDE,t} \equiv \frac{v_{GDE,t}}{q_{GDE,t}} = \frac{1}{\omega_{1,t}\dfrac{1}{p_{1,t}} + \omega_{2,t}\dfrac{1}{p_{2,t}}} \ ,$$

where $\omega_{j,t}$ is the share of good *i* in nominal GDE:

(C12) $$\omega_{i,t} \equiv \frac{p_{i,t}c_{i,t}}{v_{GDE,t}} \quad i = 1,2 \ .$$

We next turn to the main question of interest, namely how to measure real GDI. In this simple model, there are four prices one could envisage to use to deflate the nominal trade balance when computing real GDI: the price of imports, the price of exports, the GDP price deflator, or the GDE price deflator. This yields the following four measures of real GDI (the upperscript identifies the price used to deflate the trade balance: *M* for imports, *X* for exports, *P* for GDP and *E* for GDE):

$$\text{(C13)} \quad q^{M}_{GDI,t} = c_{1,t} + c_{2,t} + \frac{p_{1,t}}{p_{2,t}} x_{1,t} - m_{2,t}$$

$$\text{(C14)} \quad q^{X}_{GDI,t} = c_{1,t} + c_{2,t} + x_{1,t} - \frac{p_{2,t}}{p_{1,t}} m_{2,t}$$

$$\text{(C15)} \quad q^{P}_{GDI,t} = c_{1,t} + c_{2,t} + \frac{p_{1,t}x_{1,t} - p_{2,t}m_{2,t}}{p_{GDP,t}}$$

$$\text{(C16)} \quad q^{E}_{GDI,t} = c_{1,t} + c_{2,t} + \frac{p_{1,t}x_{1,t} - p_{2,t}m_{2,t}}{p_{GDE,t}} \quad .$$

For each one of these four measures, there is a corresponding implicit GDI price deflator:

$$\text{(C17)} \quad p^{M}_{GDI,t} \equiv \frac{v_{GDI,t}}{q^{M}_{GDI,t}} = \frac{1}{\sigma_{1,t}\frac{1}{p_{1,t}} + (\sigma_{2,t} + \sigma_{X,t} - \sigma_{M,t})\frac{1}{p_{2,t}}} = \frac{1}{\sigma_{1,t}\frac{1}{p_{1,t}} + (1-\sigma_{1,t})\frac{1}{p_{2,t}}} \quad ,$$

where the $\sigma_{i,t}$'s are the GDP shares of its components:

$$\text{(C18)} \quad \sigma_{i,t} \equiv \frac{p_{i,t}c_{i,t}}{v_{GDP,t}} \; i = 1,2; \; \sigma_{X,t} \equiv \frac{p_{1,t}x_{1,t}}{v_{GDP,t}}; \; \sigma_{M,t} \equiv \frac{p_{2,t}m_{2,t}}{v_{GDP,t}} \quad .$$

Note that $\sigma_{1,t} + \sigma_{X,t} = s_{1,t}$ and $\sigma_{2,t} - \sigma_{M,t} = s_{2,t}$. Moreover, $\sigma_{1,t} = \omega_{1,t}$ and $\sigma_{1,t} = \omega_{1,t}$ if trade is balanced since nominal GDE and GDP are equal in that case.

Similarly, we find:

$$\text{(C19)} \quad p^{X}_{GDI,t} \equiv \frac{v_{GDI,t}}{q^{X}_{GDI,t}} = \frac{1}{\left(\sigma_{1,t} + \sigma_{X,t} - \sigma_{M,t}\right)\frac{1}{p_{1,t}} + \sigma_{2,t}\frac{1}{p_{2,t}}} = \frac{1}{\left(1-\sigma_{2,t}\right)\frac{1}{p_{1,t}} + \sigma_{2,t}\frac{1}{p_{2,t}}}$$

$$\text{(C20)} \quad \begin{aligned} p^{P}_{GDI,t} &\equiv \frac{v_{GDI,t}}{q^{P}_{GDI,t}} = \frac{1}{\sigma_{1,t}\frac{1}{p_{1,t}} + \sigma_{2,t}\frac{1}{p_{2,t}} + \left(\sigma_{X,t} - \sigma_{M,t}\right)\frac{1}{p_{GDP,t}}} \\ &= \frac{1}{\left[\sigma_{1,t} + s_{1,t}\left(\sigma_{X,t} - \sigma_{M,t}\right)\right]\frac{1}{p_{1,t}} + \left[\sigma_{2,t} + s_{2,t}\left(\sigma_{X,t} - \sigma_{M,t}\right)\right]\frac{1}{p_{2,t}}} \end{aligned}$$

$$p_{GDI,t}^{E} \equiv \frac{v_{GDI,t}}{q_{GDI,t}^{E}} = \frac{1}{\sigma_{1,t}\frac{1}{p_{1,t}} + \sigma_{2,t}\frac{1}{p_{2,t}} + \left(\sigma_{X,t} - \sigma_{M,t}\right)\frac{1}{p_{GDE,t}}}$$

$$\text{(C21)} \qquad = \frac{1}{\frac{v_{GDE,t}}{v_{GDP,t}}\left(\omega_{1,t}\frac{1}{p_{1,t}} + \omega_{2,t}\frac{1}{p_{2,t}}\right) + \left(\sigma_{X,t} - \sigma_{M,t}\right)\frac{1}{p_{GDE,t}}} \quad .$$

$$= \frac{1}{(1 - \sigma_{X,t} + \sigma_{M,t})\frac{1}{p_{GDE,t}} + \left(\sigma_{X,t} - \sigma_{M,t}\right)\frac{1}{p_{GDE,t}}} = p_{GDE,t}$$

This last result is intuitively very appealing: given that nominal GDI can only be used to buy good 1 or good 2 as given by the social preferences, it seems pretty obvious that the appropriate price deflator is the price index of gross domestic expenditure as given by $p_{GDE,t}$; $p_{GDE,t}^{-1}$ thus measures the purchasing power of gross domestic income.

In our view the first three GDI implicit price deflators must be decisively rejected for being internally inconsistent. Indeed, all three are functions of the trade balance. How can it be that for a given nominal income and a given price of final demand, real income should generally be a function of the trade balance? A trade surplus or deficit is an indication of a gap between income and expenditure, but real income is predetermined by real GDP and the terms of trade: it is independent of the saving or dissaving decision. If these three price deflators must be rejected, then the corresponding measures of real GDI must be rejected as well, leaving $q_{GDI,t}^{E}$ as the only acceptable measure of real GDI in this model. This confirms our results obtained from a model that treats imports and exports as middle products. Note also that real GDI can be obtained directly as:

$$\text{(C22)} \quad q_{GDI,t}^{E} = \frac{v_{GDI,t}}{p_{GDE,t}} \ ,$$

whereas as the trading gain is obtained as:

$$\text{(C23)} \quad g_{TG,t} \equiv q_{GDP,t} - q_{GDI,t} = x_{1,t}\left(\frac{p_{1,t}}{p_{GDE,t}} - 1\right) - m_{2,t}\left(\frac{p_{2,t}}{p_{GDE,t}} - 1\right) .$$

It is noteworthy that all four approaches reviewed above yield the same result in the unlikely situation of balanced trade: real GDI is then equal to real GDE, with the implicit GDI price deflator equal to the implicit GDE price deflator in all four cases. As shown earlier, this is not so in the general case when there is a trade surplus or deficit. One might be tempted to argue that deflators (C17), (C19) and (C20) are therefore acceptable given that balanced trade might be a reasonable first approximation, in which case (C13)–(C15) are *locally* internally consistent. This is correct, but given that (C16) is *globally* internally consistent, the use of the price of final domestic expenditure as the deflator of the trade account is overwhelming.

One might argue that the model used here is unduly restrictive given that there are no transformation possibilities between outputs at the technology level, and no substitution possibilities between goods at the preference level. Admittedly, the model is kept as simple as possible, but adopting the usual form of the HOS model would not alter our conclusions. Furthermore, it is important to remember that the model used here is *exact* for the Laspeyres quantity aggregation. If one wants to do justice to the nonlinearities of the HOS model, then one should adopt superlative indices that would be capable of giving a quadratic approximation to reality. Nonetheless, this would not alter our conclusions.

Finally, one should note that since there are only two prices in this model, there is only one possible price ratio, and hence the terms-of-trade effect *is* the trading-gain effect. There is no room for a real-exchange-rate effect since there are no nontraded goods. This is just another illustration of how the HOS model departs from the treatment of the SNA where the GDE components are clearly distinct from imports and exports.

C.2 Numerical Illustration

We conclude with a simple numerical illustration of our results. For simplicity, we assume a linear (i.e. classical) version of the HOS model. This will suffices to demonstrate our main point. It is noteworthy that this linear version is *exact* for the Laspeyres quantity aggregation that is so widely used in practice. If one wants to do justice to the neoclassical properties of the HOS model, then nonlinear aggregation techniques should be used such as Cobb-Douglas (also known as geometric Laspeyres) or superlative indices, but our results would be unaltered.

Thus, we assume that the elasticity of transformation between outputs is zero: the production possibilities frontier has thus an inverted-L shape. For simplicity, we will also assume the absence of economic growth, be it as a result of increases in factor endowments or of technological change. On the demand side, we will assume that the two good are perfect complements for each other: the social indifference curves are therefore L-shaped. Moreover, we will assume that the social preferences are constant and homothetic: the Engle curve is therefore a straight line through the origin. For illustrative purposes and without any loss of generality, we assume again that the country exports the first good and imports the second one.

The initial production equilibrium (at time 0) is depicted in Figure 1 at point Y_0. Output of good 1 is equal to 3 and output of good 2 is equal to 1. The initial prices of both goods are unity; the terms of trade line is given by TOT_0, with slope -1. Assuming balanced trade, the initial consumption point is at C_0, with consumption of both goods equal to 2. Both exports of good 1 and imports of good 2 are equal to 1. Nominal and real GDP, GDI, and GDE are equal to 4, and all three deflators are unity. See Table 1, column 1 for the values of the remaining variables of interest.

We next consider a worsening in the terms of trade, with the price of the second good doubling to 2. The new terms-of-trade line is given by TOT_1, which is now has a slope of -1/2. Production equilibrium remains at Y_0 since there are no transformation possibilities, but, still assuming balanced trade, the consumption point moves to C_1, with the consumption of both goods falling to 1.67. Exports have increased by one third, whereas imports have fallen by one third. Nominal GDP, GDI (and GDE since trade is balanced) increase from 4 to 5 as the result of the increase in the price of the second good, but the GDP deflator increases in the same proportions, thus leaving real GDP unchanged. The price of gross domestic expenditure (the GDE deflator) increases more substantially, from 1 to 1.5, given the doubling in the price of the second good, implying a reduction in real GDE, from 4 to 3.33, matching the downward movement in consumption from C_0 to C_1. Real GDI is equal to real GDE, independently of which price index is used to deflate the trade balance since it is nil, and hence the GDI price deflator is equal to the GDE price deflator in all four cases.

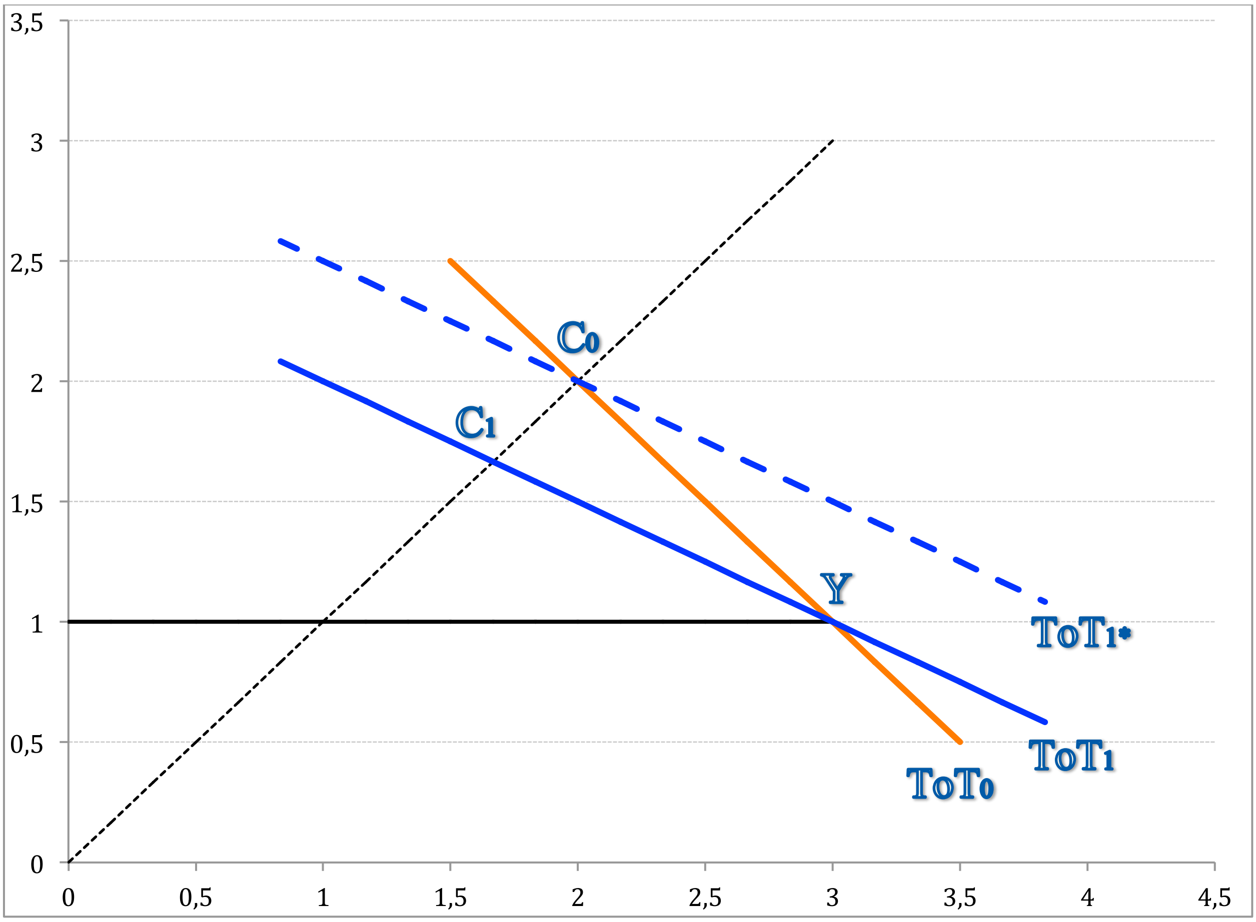
The results would be quite different, however, if we contemplated the more likely case of a trade in-balance. Given the adverse change in the terms of trade, the country might be led to run a trade deficit in order to limit the size of the adjustment in final demand, particularly so if the change in the terms of trade may be deemed to be temporary. In order not to overload the figure, simply consider what would happen if domestic demand had not adjusted at all to the change in the terms of trade, i.e. if the country had chosen to run a trade deficit in order to maintain its previous level of domestic demand. In that case the terms of trade line is given by TOT_{1*}, with the same slope as TOT_1, but going through point C_0 rather than C_1. This implies a trade balance of -1. The values of all variables in this scenario (Scenario 1*) can be found in column 3 of Table 1. Nominal GDP and GDI, at 5, are the same as in Scenario 1, but nominal GDE is obviously higher now that the quantities absorbed are larger. The price deflator of GDE, at 1.5, is the same as in Scenario 1, given that the prices that the residents face are the same; real GDE, on the other hand, at 4 is the same as in the base period since the quantities absorbed are the same. Of particular interest is the value of q_{GDI}^{E}: at 3.33 it is the same in Scenario 1* as in Scenario 1, and naturally the GDI price deflator, which is the same as the GDE price deflator, remains unchanged as well. This makes sense, since the decision to spend more than one's income should have no impact on the real value of income. The other three options, however, lead to the rather bizarre result that the decision to save or to dis-save impacts real income. Thus, deflating the trade balance by the price of imports shows an increase in real GDI from 3.33 to 3.5 in this example. Thus, this would suggest that real income increases as domestic residents decide to spend more than their income. The more you spend, the higher your income! There is no rational economic explanation behind this result, simply a case of faulty measurement. The use of the price of exports or the implicit price of GDP as the deflator of the trade account leads to the equally strange result that income falls as residents choose to spend more.

We return to the question asked at the end of the main text: what is real GDP meant to measure? Is it input, is it activity, is it output, is it production, is it real value added, is it real income? In a closed economy, and assuming away technological change, the answer would have to be "all of the above". For a given production possibilities set, assuming allocative efficiency, production and absorption will take place on its frontier, and real GDP, properly measured, will be constant along that frontier. All intermediate products are by definition nontraded and they net out. Thus, input (assuming constant return to scale), activity, output, production, real value added, real income could all be viewed as being equivalent to real GDP. Admittedly, some of these concepts are rather elusive, but we could interpret activity as TFP-augmented input (having re-introduced technological change), production as synonymous for activity or output, and real value added as equivalent to real income. If international trade in end products is allowed for, like it is in the HOS model, real GDP can no longer be viewed as identical to real income (real GDI), or to real value added depending at what stage the latter is measured, but the equivalence between the other four interpretations arguably remains valid. If trade in middle products is allowed for, then all six interpretations need to be rejected, and real GDP merely is a metric of the production possibilities frontier as argued in the main text. Noting that in reality, the overwhelming part of international trade is in middle products, the intuitive appeal of real GDP is much reduced. Moreover, given that the SNA clearly treats aggregate imports and exports as middle products rather than as end products, the central role of real GDP in the national accounts is all the harder to justify.

Table C1
Real GDI and trading gains in the HOS model: Numerical illustration

Period 0	**Period 1**	**Period 1***
Assumptions		
$p_1 = 1$	$p_1 = 1$	$p_1 = 1$
$p_2 = 1$	$p_2 = 2$	$p_2 = 2$
$p_1x_1 - p_2m_2 = 0$	$p_1x_1 - p_2m_2 = 0$	$p_1x_1 - p_2m_2 = -1$
$y_1 = 3$	$y_1 = 3$	$y_1 = 3$
$y_2 = 1$	$y_2 = 1$	$y_2 = 1$
Results		
$c_1 = 2$	$c_1 = 1.67$	$c_1 = 2$
$c_2 = 2$	$c_2 = 1.67$	$c_2 = 2$
$x_1 = 1$	$x_1 = 1.33$	$x_1 = 1$
$m_2 = 1$	$m_2 = 0.67$	$m_2 = 1$
$v_{GDP} = 4$	$v_{GDP} = 5$	$v_{GDP} = 5$
$v_{GDI} = 4$	$v_{GDI} = 5$	$v_{GDI} = 5$
$v_{GDE} = 4$	$v_{GDE} = 5$	$v_{GDE} = 6$
$q_{GDP} = 4$	$q_{GDP} = 4$	$q_{GDP} = 4$
$q_{GDE} = 4$	$q_{GDE} = 3.33$	$q_{GDE} = 4$
$p_{GDP} = 1$	$p_{GDP} = 1.25$	$p_{GDP} = 1.25$
$p_{GDE} = 1$	$p_{GDE} = 1.5$	$p_{GDE} = 1.5$
$q^M_{GDI} = 4$	$q^M_{GDI} = 3.33$	$q^M_{GDI} = 3.5$
$q^X_{GDI} = 4$	$q^X_{GDI} = 3.33$	$q^X_{GDI} = 3$
$q^P_{GDI} = 4$	$q^P_{GDI} = 3.33$	$q^P_{GDI} = 3.2$
$q^E_{GDI} = 4$	$q^E_{GDI} = 3.33$	$q^E_{GDI} = 3.33$
$p^M_{GDI} = 1$	$p^M_{GDI} = 1.5$	$p^M_{GDI} = 1.43$
$p^X_{GDI} = 1$	$p^X_{GDI} = 1.5$	$p^X_{GDI} = 1.67$
$p^P_{GDI} = 1$	$p^P_{GDI} = 1.5$	$p^P_{GDI} = 1.56$
$p^E_{GDI} = 1$	$p^E_{GDI} = 1.5$	$p^E_{GDI} = 1.5$
	$g_{TG} = 0$	$g_{TG} = $ -0.67

Figure C1
Real GDI and trading gains in the HOS model



References

Australian Bureau of Statistics (2021) *Australian System of National Accounts* (Canberra: Australian Bureau of Statistics).

Boulat, Régis (2006) "La productivité et sa mesure en France (1944-1955)", *Histoire & mesure* XXI-1, 79-110.

Bureau of Economic Analysis (2021) *Concepts and Methods of the U.S. National Income and Product Accounts* (Washingron, D.C.: U.S. Department of Commerce).

Burge, R.W. and R.C. Geary (1957) "Balancing of a System of National Accounts in Real Terms", Chapter 2 in United Nations, Economic and Social Council, *A System of Price and Quantity Indexes for National Accounts*, E/CN.3/L.46.

Burgess, David F. (1974) "Production Theory and the Derived Demand for Imports", *Journal of International Economics* 4, 103-117.

Christensen, Laurits R., Dale W. Jorgenson, and Lawrence J. Lau (1973) "Transcendental Logarithmic Production Frontiers", *Review of Economics and Statistics* 55, 28-45.

Corden, W. Max (1992) "Dependent Economy Model of the Balance of Payments", *New Palgrave Dictionary of Money and Finance* (London: Macmillan).

Denison, Edward F. (1981) "International Transactions in Measures of the Nation's Production", *Survey of Current Business* 61, May, 17-28.

Diewert, W. Erwin (1974) "Applications of Duality Theory", in Michael D. Intriligator and David A. Kendrick (eds.) *Frontiers of Quantitative Economics*, Vol. 2 (Amsterdam: North-Holland).

Diewert, W. Erwin (1976) "Exact and Superlative Index Numbers", *Journal of Econometrics* 4, 115-145.

Diewert, W. Erwin and Denis Lawrence (2006) *Measuring the Contributions of Productivity and Terms of Trade to Australia's Economic Welfare*, report by Meyrick and Associates to the Productivity Commission, Canberra.

Diewert, W. Erwin and Catherine J. Morrison (1986) "Adjusting Output and Productivity Indices for Changes in the Terms of Trade", *Economic Journal* 96, 659-679.

European Commission (2013) *European System of Accounts, ESA 2010* (Luxembourg: Publications Office of the European Union).

Grimes, Arthur and Shine Wu (2022) "Sustainable consumption growth: New Zealand's surprisingperformance", *New Zealand Economic Papers*, DOI:10.1080/00779954.2022.2138517.

Hall, Alan (2011) "Gross Domestic Product and the Terms of Trade", *The Australian Economic Review* 44, 245-57.

International Monetary Fund (2009a) *System of National Accounts 2008* (New York: International Monetary Fund).

International Monetary Fund (2009b) *Export and Import Price Index Manual: Theory and Practic*e (Washington, D.C.: International Monetary Fund).

Kohli, Ulrich (1978) "A Gross National Product Function and the Derived Demand for Imports and Supply of Exports", *Canadian Journal of Economics* 11, 167-182.

Kohli, Ulrich (1983) "Technology and the Demand for Imports", *Southern Economic Journal* 50, 137-150.

Kohli, Ulrich (1990) “Growth Accounting in the Open Economy: Parametric and Nonparametric Estimates”, *Journal of Economic and Social Measurement* 16, 125-136.

Kohli, Ulrich (1991) *Technology, Duality, and Foreign Trade: The GNP Function Approach to Modeling Imports and Exports* (Ann Arbor, MI: University of Michigan Press).

Kohli, Ulrich (2004a) “Real GDP, Real Domestic Income, and Terms-of-Trade Changes”, *Journal of International Economics* 62, 83-106.

Kohli, Ulrich (2004b) “An Implicit Törnqvist Index of Real GDP”, *Journal of Productivity Analysis* 21, 337-353.

Kohli, Ulrich (2005) “Switzerland’s Growth Deficit: A Real Problem – But Only Half as Bad as it Looks”, in Lukas Steinmann und Hans Rentsch (eds.) *Diagnose: Wachstumsschwäche* (Zurich: Verlag Neue Zürcher Zeitung).

Kohli, Ulrich (2006a) “Terms of Trade, Real Exchange Rates, and Trading Gains”, SSHRC Conference on Price and Productivity Measurement, Vancouver, B.C., Canada, July 2004.

Kohli, Ulrich (2006b) “Real GDP, Real GDI, and Trading Gains: Canada, 1981-2005”, *International Productivity Monitor* 13, 46-56.

Kohli, Ulrich (2007) “Terms-of-Trade Changes, Real GDP, and Real Value Added in the Open Economy: Reassessing Hong Kong's Growth Performance”, *Asia-Pacific Journal of Accounting & Economics* 14, 87-109.

Kohli, Ulrich (2010) “Labour Productivity: Average vs. Marginal”, in W. Erwin Diewert, Bert M. Balk, Dennis Fixler, Kevin J. Fox, and Alice O. Nakamura (eds.) *Price and Productivity Measurement*, Volume 6 (Victoria, B.C.: Trafford Press).

Kohli, Ulrich (2022) “Trading Gains and Productivity: A Törnqvist Approach”, *International Productivity Monitor* 42, 63-86.

Kohli, Ulrich (2023) “Trading Gains: Terms-of-Trade and Real Exchange-Rate Effects”, *Review of Income and Wealth*, forthcoming.

Reinsdorf, Marshall B. (2010) “Terms of Trade Effects: Theory and Measurement”, *Review of Income and Wealth* 56, S177-S205.

Salter, W.E.G. (1959) “Internal and External Balance: The Role of Price and Expenditure Effects”, *Economic Record* 35, 226-238.

Sanyal, K.K., Jones, R.W. (1982) “The Theory of Trade in Middle Products”, *American Economic Review* 72, 16-31.

Silver, Mick and Khashayar Mahdavy (1989) “The Measurement of a Nations’s Terms of Trade Effect and Real National Disposable Income within a National Accounting Framework”, *Journal of the Royal Statistical Society*, Series A, 87-107.

Statistics Canada (2016) *User Guide: Canadian System of Macroeconomic Accounts* (Ottawa, Ontario: Statistics Canada).

Swan, Peter L. (2022) “Why ABS Volume GDP Figures Can Be Misleading”, University of New South Wales, manuscript.

Swiss National Bank (2007) “Gross domestic product, trading gains and gross national income”, *SNB Monthly Statistical Bulletin*, July.

Woodland, Alan D. (1982) *International Trade and Resource Allocation* (Amsterdam: North-Holland).