Real GDI, Trading Gains, and Productivity

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Outline

1. Introduction; the national accounts identity; the many faces of real GDI; imports and exports as middle products.

2. Real national-accounting in the Laspeyres case; consistent measure of real GDI and of the trading gains, together with their terms-of-trade and real exchange-rate components.

3. The use of the price of imports as a deflator of the trade account when deriving the trading gains and real GDI – like it is being done by the ABS – leads to an internal inconsistency and hence must be decisively rejected.

4. Real national-accounting in the Törnqvist case; the trading-gain index can again be formally decomposed into terms-of-trade and real-exchange-rate components.

Outline, continued

5. Real GDI is often a more relevant reference than real GDP when deriving measures of productivity, since trading gains are generally acquired during production, rather than after. Moreover, the distinction between trading and productivity gains can be somewhat blurred in many cases; this speaks in favor of considering both these sources of income growth jointly.

6. Conclusion: the case for using the price of gross domestic final expenditure as the deflator of nominal GDI is overwhelming.

All variables are fully defined in the paper; Appendices A, B, and C provide supporting material.

1. Introduction

What is **Gross Domestic Income** (GDI)?

Most first-year economics students know that (nominal) GDI is equal to the country's wage bill, plus the gross operating surplus, plus indirect taxes, minus subsidies.

They also know that except for a statistical discrepancy, (nominal) GDI should be equal to (nominal) **Gross Domestic Product** (GDP).

Neglecting the statistical discrepancy and net indirect taxes for simplicity, one can thus write (the *v*'s denote nominal figures):

(1) $$v_{GDP,t} \equiv v_{C,t} + v_{I,t} + v_{G,t} + v_{X,t} - v_{M,t} = v_{L,t} + v_{K,t} \equiv v_{GDI,t} \ .$$

Yet, if these students should search for a dollar figure of GDI on the U.S. Bureau of Economic Analysis (BEA) website, they would find none. They would almost invariably be directed to GDP. The best they might find is a ***reference*** to the ***gross-domestic-income measure*** of GDP.

They would be luckier if they searched for a related – but distinct – concept, namely (nominal**) Gross Domestic Expenditure** (GDE), defined as GDI minus the trade balance:

(2) $$v_{GDE,t} \equiv v_{GDI,t} - (v_{X,t} - v_{M,t}) = v_{C,t} + v_{I,t} + v_{G,t} \ .$$

The many faces of real Gross Domestic Income (GDI)

While is difficult to find any trace of *nominal* GDI in official statistics publications, there are a plethora of competing definitions of *real* GDI that one can find there and in the literature:

i) Nominal GDI deflated by the implicit GDP price deflator
ii) Command-basis GDP, often interpreted as real GDI
iii) Real GDP plus the trading gains (knowing that there are at least ten different ways to calculate the trading gains, this yields at a minimum ten more measures of real GDI)
iv) Nominal GDI deflated by the price of gross domestic final expenditure

It is interesting to note that all but the last of these definitions rely on real GDP or its deflator to define real GDI.

Production-theory approach to modeling imports and exports

Most international trade is in raw materials and intermediate goods

Even most so-called finished products that are traded are not ready to meet final demand: they must typically still go through a number of costly transformations at home or abroad during which they get combined with local labor and capital services, so that a significant part of their final price tag typically accounted for by local value added.

This treatment of imports and exports as middle products is consistent with the approach implicit in the SNA that divides final demand into three main components, which, can be viewed as *de facto* nontraded goods; imports and exports are treated separately.

Nonetheless, our approach is also fully valid if traded goods are viewed as end products such as in the Heckscher-Ohlin-Samuelson model; see Appendix C.

2. Real National-Accounting in the Laspeyres Case

Nominal GDP:

$$(3) \qquad v_{GDP,t} \equiv v_{C,t} + v_{I,t} + v_{G,t} + v_{X,t} - v_{M,t} = p_{C,t}q_{C,t} + p_{I,t}q_{I,t} + p_{G,t}q_{G,t} + p_{X,t}q_{X,t} - p_{M,t}q_{M,t} \ .$$

Laspeyres quantity index of **real GDP**:

$$(4) \qquad q_{GDP,t} \equiv \frac{v_{C,t}}{p_{C,t}} + \frac{v_{I,t}}{p_{I,t}} + \frac{v_{G,t}}{p_{G,t}} + \frac{v_{X,t}}{p_{X,t}} - \frac{v_{M,t}}{p_{M,t}} = q_{C,t} + q_{I,t} + q_{G,t} + q_{X,t} - q_{M,t} \ .$$

Implicit **GDP price** deflator (a Paasche price index):

$$(5) \qquad p_{GDP,t} \equiv \frac{v_{GDP,t}}{q_{GDP,t}} = \frac{1}{s_{C,t}\frac{1}{p_{C,t}} + s_{I,t}\frac{1}{p_{I,t}} + s_{G,t}\frac{1}{p_{G,t}} + s_{X,t}\frac{1}{p_{X,t}} - s_{M,t}\frac{1}{p_{M,t}}} \ .$$

Real GDE and real GDI

Laspeyres quantity index of **real GDE**: (6) $$q_{GDE,t} \equiv \frac{v_{C,t}}{p_{C,t}} + \frac{v_{I,t}}{p_{I,t}} + \frac{v_{G,t}}{p_{G,t}} = q_{C,t} + q_{I,t} + q_{G,t}$$

Implicit **GDE price** deflator, i.e. the implicit **price of gross domestic final expenditure**:

(7) $$p_{GDE,t} \equiv \frac{v_{GDE,t}}{q_{GDE,t}} = \frac{1}{\omega_{C,t}\frac{1}{p_{C,t}} + \omega_{I,t}\frac{1}{p_{I,t}} + \omega_{G,t}\frac{1}{p_{G,t}}} \ .$$

$p_{GDE,t}$ is the price of what domestic residents buy; its inverse, $p_{GDE,t}^{-1}$, is therefore the obvious indicator of the purchasing power of nominal GDI; thus, **real GDI** is obtained as:

(8) $$q_{GDI,t} \equiv \frac{v_{GDI,t}}{p_{GDE,t}} = \frac{v_{GDE,t}}{p_{GDE,t}} + \frac{v_{X,t} - v_{M,t}}{p_{GDE,t}} = q_{GDE,t} + q_{X,t}\frac{p_{X,t}}{p_{GDE,t}} - q_{M,t}\frac{p_{M,t}}{p_{GDE,t}} \ .$$

Trading gains

Applying the SNA and IMF definition of real GDI in reverse, the **trading gains** are:

$$(9) \qquad g_{TG,t} \equiv q_{GDI,t} - q_{GDP,t} = q_{X,t}\left(\frac{p_{X,t}}{p_{GDE,t}} - 1\right) - q_{M,t}\left(\frac{p_{M,t}}{p_{GDE,t}} - 1\right) = v_{GDI,t}\left(\frac{1}{p_{GDE,t}} - \frac{1}{p_{GDP,t}}\right) .$$

The **real trade balance** can be denoted for short as:

$$(10) \qquad b_{TB,t} \equiv \frac{v_{X,t} - v_{M,t}}{p_{GDE,t}} .$$

The **national accounts identity** can therefore be expressed in real terms as:

$$(11) \qquad q_{GDP,t} + g_{TG,t} = q_{GDI,t} = q_{GDE,t} + b_{TB,t} .$$

Indices of factor quantities and rental prices

Nominal GDI, defined by (1), can also be written as:

(12) $$v_{GDI,t} \equiv v_{L,t} + v_{K,t} = w_{L,t} x_{L,t} + w_{K,t} x_{K,t} \ .$$

The Laspeyres index of **real factor services** (input quantities) is given by:

(13) $$x_{GDI,t} = x_{L,t} + x_{K,t} \ ,$$

and $w_{GDI,t}$, the implicit **user cost deflator** (input prices), then has the Paasche form:

(14) $$w_{GDI,t} \equiv \frac{v_{GDI,t}}{x_{GDI,t}} = \frac{1}{\sigma_{L,t} \frac{1}{w_{L,t}} + \sigma_{K,t} \frac{1}{w_{K,t}}} \ .$$

Total factor productivity

In analogy to (4) and (5), (13) and (14) could be viewed as indices of real GDI (yet another definition of real GDI) and $w_{GDI,t}$ in (14) as the implicit GDI price deflator.

One must keep in mind, however, that we have adopted $p_{GDE,t}^{-1}$ as our measure of purchasing power, and hence one must realize that $w_{GDI,t}$ contains a real element if domestic factors become more productive over time.

This leads to the definition of the Laspeyres index of **total factor productivity** (TFP) as a Solow residual:

$$(15) \qquad r_{TFP,t} \equiv q_{GDP,t} - x_{GDI,t} = v_{GDP,t}\left(\frac{1}{p_{GDP,t}} - \frac{1}{w_{GDI,t}}\right) .$$

The terms of trade and the real exchange rate

As shown in Appendix A, it is possible to decompose the trading-gain index into a terms-of-trade component and a real-exchange-rate component.

The **terms of trade** (h_t), are defined as: (16) $$h_t \equiv \frac{p_{X,t} - p_{M,t}}{p_{GDE,t}}$$

The **price of traded goods** ($p_{T,t}$), is defined as: (17) $$p_{T,t} \equiv \frac{1}{2} p_{X,t} + \frac{1}{2} p_{M,t}$$

The **real exchange rate** (e_t), then is: (18) $$e_t \equiv \frac{p_{T,t} - p_{GDE,t}}{p_{GDE,t}} \ .$$

Decomposition of the trading gain

The **trading-gain index** (9) can then be written as:

(19) $$g_{TG,t} = g_{ToT,t} + g_{RER,t} \ ,$$

where, the **terms-of-trade effect**, can be formally derived as (see Appendix A):

(20) $$g_{ToT,t} \equiv \frac{1}{2}\left(q_{X,t} + q_{M,t}\right)h_t$$

and the **real-exchange-rate effect** is found to be:

(21) $$g_{RER,t} \equiv \left(q_{X,t} - q_{M,t}\right)e_t$$

In summary

In the Laspeyres case, the complete decompositions of real GDP, GDI and GDE therefore are:

Real GDP: (22) $q_{GDP,t} = x_{GDI,t} + r_{TFP,t}$

Real GDI: (23) $q_{GDI,t} = x_{GDI,t} + r_{TFP,t} + g_{ToT,t} + g_{RER,t}$

Real GDE: (24) $q_{GDE,t} = x_{GDI,t} + r_{TFP,t} + g_{ToT,t} + g_{RER,t} - b_{TB,t}$.

3. Using the Price of Imports as the Deflator of the Trade Account

This is the approach used by the BEA until 2010, and by most national statistical agencies around the world, including the Australian Bureau of Statistics (ABS), even today (the use of all but one of the other indices suggested by the SNA leads to the same dead end).

The use of the price of imports as the deflator of the trade account leads to the **ABS estimate of real GDI** (or of command-basis GDP to use the BEA terminology):

$$(25) \qquad q_{GDI,t}^{ABS} \equiv q_{C,t} + q_{I,t} + q_{G,t} + \frac{v_{X,t} - v_{M,t}}{p_{M,t}} = q_{GDE,t} + q_{X,t}\frac{p_{X,t}}{p_{M,t}} - q_{M,t} \ .$$

Thus, the only difference between real GDI thus defined and real GDP as given by (3) is the use in (25) of the price of imports to deflate nominal exports.

The ABS approach

Using the import price as a deflator, the estimate of the trading gains is as follows:

$$(26) \qquad g_{TG,t}^{ABS} \equiv q_{GDI,t}^{ABS} - q_{GDP,t} = q_{X,t}\left(\frac{p_{X,t}}{p_{M,t}} - 1\right) .$$

The implicit GDI price deflator then is:

$$(27) \qquad p_{GDI,t}^{ABS} \equiv \frac{v_{GDI,t}}{q_{GDI,t}^{ABS}} = \frac{1}{s_{GDE,t}\frac{1}{p_{GDE,t}} + (s_{X,t} - s_{M,t})\frac{1}{p_{M,t}}} .$$

This price index is internally inconsistent since it implies that a change in import prices would generally affect real income *for a given nominal income and a given price of final expenditure!*

The resulting inconsistency in the national accounts

The use of the price of imports as the deflator of the trade account leads to an inconsistency in the national accounts. This inconsistency is identified by the following **error term**:

$$(28) \qquad \Delta_t^{ABS} \equiv q_{GDI,t} - q_{GDI,t}^{ABS} = g_{TG,t} - g_{TG,t}^{ABS} = \left(v_{X,t} - v_{M,t}\right)\left(\frac{1}{p_{GDE,t}} - \frac{1}{p_{M,t}}\right) .$$

Δ_t^{ABS} is a measure of the inconsistency of the approach of the ABS and others. It demonstrates that the measure of the trading gains given by (26) is generally incomplete because it ignores the effect of the change in import prices relative to the prices of domestic goods.

This inconsistency also appears in plain sight in the **national accounts identity**:

$$(29) \qquad q_{GDE,t} + b_{TB,t} = q_{GDI,t}^{ABS} + \Delta_t^{ABS} = q_{GDP,t} + g_{TG,t}^{ABS} + \Delta_t^{ABS} .$$

In summary

The approach used by most national statistical agencies – with the notable exceptions of Statistics Canada and today's BEA – leads to a GDI deflator that is generally internally inconsistent. The corresponding measure of real GDI must therefore be viewed as flawed and must be rejected as well.

Furthermore, the resulting measures of the trading gains are at best (under balanced trade) measures of the terms-of-trade effect and they are therefore incomplete and misnamed.

One way to deal with this inconsistency, short of correcting it, is to simply ignore it, which probably goes a long way in explaining why real GDI even today is still treated as somewhat of an afterthought, without really been integrated in the national accounts framework.

4. The Törnqvist Approach

Let $V_{GDP,t,t-1} \equiv v_{GDP,t} \big/ v_{GDP,t-1}$ be the **growth factor of nominal GDP** (or, equivalently, nominal GDI); Diewert and Morrison (1986) show that it can be expressed as:

(30) $V_{GDP,t,t-1} = P_{GDP,t,t-1} \cdot X_{GDI,t,t-1} \cdot R_{TFP,t,t-1}$ where

(31) $P_{GDP,t,t-1} \equiv \left(\frac{p_{X,t}}{p_{X,t-1}} \right)^{\bar{s}_{X,t}} \left(\frac{p_{M,t}}{p_{M,t-1}} \right)^{-\bar{s}_{M,t}} \left(\frac{p_{GDE,t}}{p_{GDE,t-1}} \right)^{\bar{s}_{GDE,t}}$ is a Törnqvist **output price index**,

(32) $X_{GDI,t,t-1} \equiv \left(\frac{x_{L,t}}{x_{L,t-1}} \right)^{\bar{\sigma}_{L,t}} \left(\frac{x_{K,t}}{x_{K,t-1}} \right)^{\bar{\sigma}_{K,t}}$ is a Törnqvist **input quantity index**

Diewert and Morrison (1986) show that these indices are exact if the underlying GDP function is Translog.

Real GDP

$R_{TFP,t,t-1}$ is a the **TFP growth factor** and it is obtained as usual as a residual:

(33) $$R_{TFP,t,t-1} \equiv V_{GDP,t,t-1} \cdot P^{-1}_{GDP,t,t-1} \cdot X^{-1}_{GDI,t,t-1} \ .$$

Both $X_{GDI,t,t-1}$ and $R_{TFP,t,t-1}$ are real growth factors and their product yields the **real-GDP growth factor**:

(34) $$Q_{GDP,t,t-1} \equiv V_{GDP,t,t-1} \big/ P_{GDP,t,t-1} = X_{GDI,t,t-1} \cdot R_{TFP,t,t-1} \ .$$

Real GDI and Trading Gains

Next, in analogy to (8), we obtain the **real-GDI growth factor**:

(35) $$Q_{GDI,t,t-1} \equiv V_{GDI,t,t-1} \big/ P_{GDE,t,t-1} \ .$$

The **trading-gain growth factor** can then be obtained in the same way as in (9):

(36) $$G_{TG,t,t-1} \equiv Q_{GDI,t,t-1} \big/ Q_{GDP,t,t-1} = P_{GDP,t,t-1} \,/\, P_{GDE,t,t-1} \ .$$

This shows that the trading gains can be obtained simply by taking the ratio of two price indices widely available in the national account statistics.

Terms of trade and real exchange rate re-defined

The **terms of trade** (h_t) is now re-defined in the traditional way:

$$(37) \qquad h_t \equiv \frac{p_{X,t}}{p_{M,t}} \ .$$

The **price of traded goods** ($p_{T,t}$) is re-defined as the geometric average of the prices of exports and imports:

$$(38) \qquad p_{T,t} \equiv p_{X,t}^{1/2} p_{M,t}^{1/2} \ .$$

Finally, the **real exchange rate** (e_t) is also re-defined in the traditional way:

$$(39) \qquad e_t \equiv \frac{p_{T,t}}{p_{GDE,t}} = \frac{p_{X,t}^{1/2} p_{M,t}^{1/2}}{p_{GDE,t}} \ .$$

Decomposition of the trading gain

The **trading-gain factor** can be decomposed as follows:

(40) $G_{TG,t,t-1} \equiv G_{ToT,t,t-1} \cdot G_{RER,t,t-1}$ where

(41) $$G_{ToT,t,t-1} \equiv \left(\frac{h_t}{h_{t-1}} \right)^{(\bar{s}_{X,t}+\bar{s}_{M,t})/2} = \left(\frac{p_{X,t}}{p_{X,t-1}} \right)^{(\bar{s}_{X,t}+\bar{s}_{M,t})/2} \left(\frac{p_{M,t}}{p_{M,t-1}} \right)^{-(\bar{s}_{X,t}+\bar{s}_{M,t})/2}$$

measures the **terms-of-trade effect**, and

(42) $$G_{RER,t,t-1} \equiv \left(\frac{e_t}{e_{t-1}} \right)^{\bar{s}_{X,t}-\bar{s}_{M,t}} = \left(\frac{p_{X,t}}{p_{X,t-1}} \right)^{(\bar{s}_{X,t}-\bar{s}_{M,t})/2} \left(\frac{p_{M,t}}{p_{M,t-1}} \right)^{(\bar{s}_{X,t}-\bar{s}_{M,t})/2} \left(\frac{p_{GDE,t}}{p_{GDE,t-1}} \right)^{-(\bar{s}_{X,t}-\bar{s}_{M,t})}$$

is the **real-exchange-rate effect**. As shown in Appendix A, this decomposition is exact if the underlying GDP function has the Translog form.

In summary

The **national accounts relationships** can be expressed in real terms in the Törnqvist case as:

(43) $$Q_{GDP,t,t-1} = X_{GDI,t,t-1} \cdot R_{TFP,t,t-1}$$

(44) $$Q_{GDI,t,t-1} = X_{GDI,t,t-1} \cdot R_{TFP,t,t-1} \cdot G_{ToT,t,t-1} \cdot G_{RER,t,t-1} = Q_{GDP,t,t-1} \cdot G_{ToT,t,t-1} \cdot G_{RER,t,t-1}$$

(45) $$Q_{GDE,t,t-1} = X_{GDI,t,t-1} \cdot R_{TFP,t,t-1} \cdot G_{TG,t,t-1} \cdot B_{TB,t,t-1}^{-1} = Q_{GDP,t,t-1} \cdot G_{TG,t,t-1} \cdot B_{TB,t,t-1}^{-1} = Q_{GDI,t,t-1} \cdot B_{TB,t,t-1}^{-1} \ ,$$

where $B_{TB,t,t-1}$ is a measure of the **trade-balance effect**:

(46) $$B_{TB,t,t-1} \equiv \frac{Q_{GDI,t,t-1}}{Q_{GDE,t-1}} \approx \left(\frac{V_{X,t,t-1}}{P_{GDE,t,t-1}} \right)^{\overline{s}_{X,t}} \left(\frac{V_{M,t,t-1}}{P_{GDE,t,t-1}} \right)^{-\overline{s}_{M,t}} Q_{GDE,t,t-1}^{-(\overline{s}_{X,t} - \overline{s}_{M,t})} = \left(\frac{V_{X,t,t-1}}{V_{GDE,t,t-1}} \right)^{\overline{s}_{X,t}} \left(\frac{V_{M,t,t-1}}{V_{GDE,t,t-1}} \right)^{-\overline{s}_{M,t}}$$

Total Factor Productivity and Trading Gains in the price space

Finally, in analogy to (14), we can define the **input price index** as:

(47) $$W_{GDI,t,t-1} \equiv V_{GDI,t,t-1} \big/ X_{GDI,t,t-1} \quad .$$

Making use of (34), it can be seen that (43) and (44) can also be expressed in the **dual price space** as:

(48) $$P_{GDP,t,t-1} = W_{GDI,t,t-1} \cdot R^{-1}_{TFP,t,t-1}$$

(49) $$P_{GDE,t,t-1} = W_{GDI,t,t-1} \cdot R^{-1}_{TFP,t,t-1} \cdot G^{-1}_{ToT,t,t-1} \cdot G^{-1}_{RER,t,t-1} = P_{GDP,t,t-1} \cdot G^{-1}_{ToT,t,t-1} \cdot G^{-1}_{RER,t,t-1} \quad .$$

5. Trading gains and productivity

Total factor productivity

For a given change in the endowment of domestic factors as given by $X_{GDI,t,t-1}$, *if properly measured*, $R_{TFP,t,t-1}$ is fully determined and thus independent of $G_{ToT,t,t-1}$ and $G_{RER,t,t-1}$.

The measurement of TFP does not depend on whether or not trading gains have been taken into account. The trading gains are simply a benefit in addition to increases in TFP.

Average labor productivity

A second measure of productivity is the average productivity of labor, i.e. the real value added per unit of labor. This is the preferred measure of productivity for many commentators and statistical agencies, including the U.S. Bureau of Labor Statistics (BLS).

The singling out of labor is somewhat problematic and needs a justification. In fact, there is no reason to impute productivity and trading gains to labor, as opposed to capital, or both. At best, one can view average labor productivity as a convenient shortcut to relate the overall performance of the economy to the work effort: labor is then used as a metric, so to speak.

The wide acceptance of this somewhat Marxist concept probably has to do in parts with early adoption by the Organisation for European Economic Co-operation (OEEC, the ancestor of the OECD) in 1949 under the influence of Jean Fourastié.

Average labor productivity: which numerator?

Average labor productivity can be defined with respect to real GDP and with respect to real GDI. The latter is to be preferred given that international trade takes place overwhelmingly in middle products, and thus occurs during the production process rather than afterwards.

An improvement in the terms of trade could reflect a refinement in the quality of exports that is not fully captured by the export price and quantity indices, which could lead to an underestimation of real GDP per unit of labor.

An improvement in the terms of trade could also be the result of a prospecting activity or of a marketing effort. To the extent that significant quantities of domestic labor and capital are diverted from domestic production to such activities, average labor productivity (and TFP) could be underestimated.

Taking the trading gains into account might help to correct for these types of biases.

The trade technology

As already stressed, almost all trade takes place during production, rather than after. In our view the "trade technology", which "transforms" exports into imports, should therefore be treated as an essential element of the country's all-embracing technology.

Because it may be difficult in many situations to clearly label what is capital deepening, what is technological progress, what is human capital enhancement, and what are pure trading gains, the line between these concepts tends to be blurred in an integrated world.

Given the risk that as a result of measurement errors one development may be wrongly imputed to one or another growth factor speaks in favor of considering all of them jointly.

See Appendix B below for a very simple example where the distinction between total factor productivity and trading gains is rather fuzzy.

The "globalized" version of average labor productivity

Let $a_{GDI,t} \equiv q_{GDI,t} / x_{L,t}$ be real GDI per unit of labor, or, in terms of growth factors:

(50) $$A_{GDI,t,t-1} \equiv Q_{GDI,t,t-1} \cdot X^{-1}_{L,t,t-1} \ .$$

It follows from (44) that this can be expressed as:

(51) $$A_{GDI,t,t-1} = G_{TG,t,t-1} \cdot X_{GDI,t,t-1} \cdot R_{TFP,t,t-1} \cdot X^{-1}_{L,t,t-1} \ .$$

Making use of (32), we find that:

(52) $$X_{GDI,t,t-1} \cdot X^{-1}_{L,t,t-1} = \left(\frac{x_{K,t}}{x_{L,t}}\right)^{\bar{\sigma}_{K,t}} \left(\frac{x_{L,t}}{x_{L,t-1}}\right)^{\bar{\sigma}_{L,t}-1} = \left(\frac{x_{K,t}/x_{L,t}}{x_{K,t-1}/x_{L,t-1}}\right)^{\bar{\sigma}_{K,t}} = \left(\frac{k_t}{k_{t-1}}\right)^{\bar{\sigma}_{K,t}} \equiv K_{t,t-1} \ .$$

The complete decomposition of both measures of average labor productivity

The complete Törnqvist decomposition of growth in the *globalized* version of domestic average labor productivity:

(53) $$A_{GDI,t,t-1} = G_{TG,t,t-1} \cdot K_{t,t-1} \cdot R_{TFP,t,t-1} = G_{ToT,t,t-1} \cdot G_{RER,t,t-1} \cdot K_{t,t-1} \cdot R_{TFP,t,t-1} \ .$$

Note that it follows from (43) and (52) that the product of the last two components yields the growth in the average labor productivity defined with respect to real GDP, $A_{GDP,t,t-1}$, or put another way, the average productivity of labor in a *closed-economy* setting:

(54) $$A_{GDP,t,t-1} \equiv Q_{GDP,t,t-1} \cdot X^{-1}_{L,t,t-1} = X_{GDI,t,t-1} \cdot R_{TFP,t,t-1} \cdot X^{-1}_{L,t,t-1} = K_{t,t-1} \cdot R_{TFP,t,t-1} \ .$$

The real marginal product of labor

As far as workers are concerned, their *marginal* product is undoubtedly of more interest than their *average* product.

Under perfect competition and optimization, the real marginal product of labor can readily be observed as the real wage rate, $u_{L,t} \equiv w_{L,t} / p_{GDE,t}$, i.e. the nominal wage deflated by the price of domestic final goods, the GDI price deflator.

Note that the nominal wage is an income concept and it therefore would make little sense to use the price of GDP as given by (31) to deflate nominal wages. Domestic residents buy domestic final goods, they do not purchase imports or exports.

Complete decomposition of the marginal product of labor

Recall that $\sigma_{L,t} \equiv (x_{L,t} w_{L,t}) \big/ v_{GDI,t} = (x_{L,t} w_{L,t}) \big/ (q_{GDI,t} p_{GDE,t})$; it follows that $u_{L,t} = a_{GDI,t}\, \sigma_{L,t}$, i.e.:

(55) $U_{L,t,t-1} = A_{GDI,t,t-1} \cdot \Sigma_{L,t,t-1}$, where

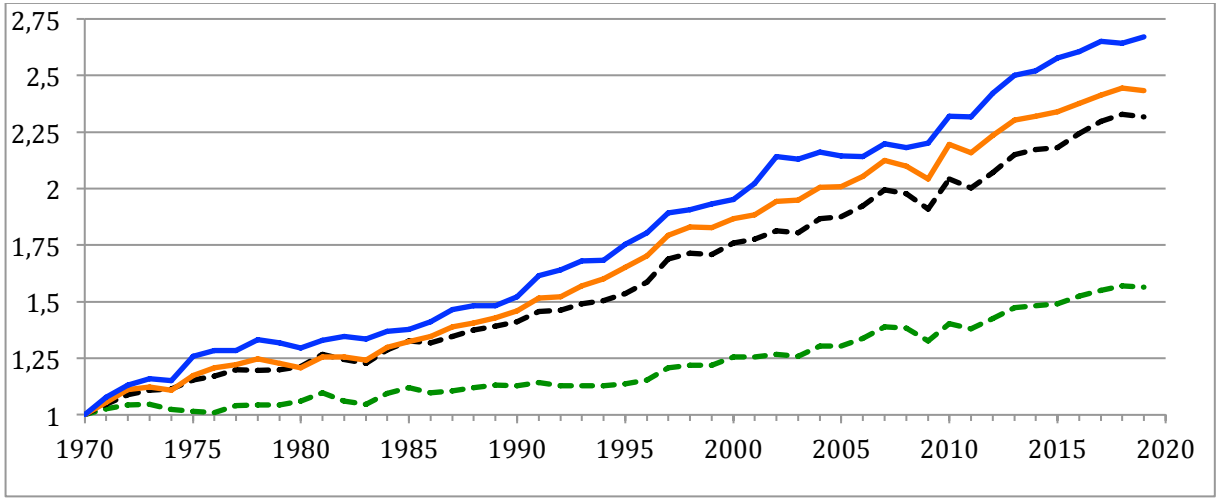
(56) $U_{L,t,t-1} \equiv u_{L,t} / u_{L,t-1}$ and

(57) $\Sigma_{L,t,t-1} \equiv \sigma_{L,t} / \sigma_{L,t-1}$.

Using (53), one gets a complete decomposition of the growth of the marginal product of labor:

(58) $U_{L,t,t-1} = \Sigma_{L,t,t-1} \cdot G_{ToT,t,t-1} \cdot G_{RER,t,t-1} \cdot K_{t,t-1} \cdot R_{TFP,t,t-1}$.

Decomposition of the marginal product of labor: Switzerland, 1970-2019



Cumulated series, from bottom to top: $R_{TFP,t,t-1} \cdot K_{t,t-1} \cdot G_{TG,t,t-1} \cdot \Sigma_{t,t-1} = U_{L,t,t-1}$

6. Conclusion

It is disappointing that the IMF, the OECD, EuroStat, and the United Nations, among others, do not have the resolution to make explicit recommendations concerning the appropriate trade-balance deflator, basically leaving member countries in the dark as to what the best practices are.

Unless trade happens to be balanced, all the so-called measures of the trading gains using a deflator other than $p_{GDE,t}$ are incomplete since they exclude the relative-price effect resulting from a change in the price of the chosen trade-account deflator relative to the price of domestic final goods. This is why additional components such as $G_{RER,t,t-1}$ are needed.

Thus, these official measures are misnamed: they should be viewed at best as measures of the terms-of-trade effects, rather than of the full trading gains. Consequently, the corresponding real GDI estimates must be considered as flawed.

Getting it backwards

It would appear that most statistical agencies get it backwards. They select a deflator, more or less at random, receiving no strict guidance from the SNA. They then very carefully calculate the (incomplete) trading gain, add it to their estimate of real GDP, and declare it to be real GDI. The implicit GDI deflator is then almost meaningless since it will generally be a function of the prices of imports and/or exports, incorrectly suggesting that a change in the prices of traded goods would change real domestic income for a *given nominal domestic income and a given domestic price level*.

Real GDI then becomes some kind of curiosity in the system of national accounts, with no obvious link to the other aggregates. Instead, these agencies and the authors of the SNA should begin by asking themselves what real GDI is supposed to measure. In our view, the obvious answer is the real purchasing power that is available domestically, at price $p_{GDE,t}$.

The Burge and Geary (1957) question settled at long last?

Once that nominal GDP has been deflated by the price of gross domestic expenditure to yield real GDI, it is straightforward to compute the trading gain by (9) in the Laspeyres case or (36) in the Törnqvist case, as a difference (or ratio) of quantities or inverted prices.

Defining real GDI in that way implies that the trade balance must be deflated by the gross final expenditure when computing real GDI in the Laspeyres case as shown by (8).

The more than six-decade old question as to what price index should be used to deflate the trade balance would then be answered once for all. A trade surplus is deferred absorption; it therefore should be clear that to express any trade disequilibrium in real terms the nominal trade balance should be deflated by the price of domestic absorption.

The trading gain can furthermore be decomposed into terms-of-trade and real-exchange-rate effects as shown above.

What is real GDP meant to measure?

Real GDP is undoubtedly one of the economic variables the most scrutinized and referred to in practice, by economists, policy makers, and the public at large. Yet, what is real GDP meant to measure? Is it input, is it activity, is it output, is it real value added, is it real income?

Our answer would have to be: none of the above!

In our opinion, under optimization and perfect competition, real GDP can probably be best viewed as being a metric of the country's domestic production possibilities frontier (PPF).

Shifts in the PPF can be explained by changes in domestic factor endowments and in TFP.

Whether this rather abstract interpretation of the meaning of real GDP justifies its widespread use by economists and non-economists alike remains an open question. Real GDI, on the other hand, should be straightforward to understand for everyone.

Why not do the obvious?

Why are the authors of the SNA so reluctant to do the obvious? Why does the IMF define real GDI as real GDP plus the trading gain, however measured, when historically – and logically – the definitional link between these two concepts went in the opposite direction?

Could it be the refusal of one large member country to give up its hold on its antiquated and somewhat mercantilist terminology of "command-basis GNP" and its own bizarre definition of real GDI as nominal GDI deflated by the implicit ***GDP*** price index?

It would be time to move on and adopt a definition of real GDI that makes sense, indeed that is intuitively obvious, namely the domestic purchasing power of nominal GDI. As suggested above, the fact that real GDI is measured in practice in all kinds of strange and arbitrary ways is probably the main reason why it has never been recognized as the major macroeconomic variable it really is.

Thank you for your attention!